6th Podlasie Conference on Mathematics

6PCM

1-4 July 2014

Białystok, Poland

Book of Abstracts

Ministry of Science and Higher Education

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The Sixth Podlasie Conference on Mathematics

July 1–4, 2014, Białystok, Poland

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List of sessions and their organizers:

Plenary Session
Algebra (Ryszard Mazurek)
Computer Science (Magdalena Kacprzak)
Computer-Assisted Formalization of Mathematics – In Memoriam of Andrzej Trybulec (Artur Korniłowicz)
Control Theory and Dynamical Systems (Zbigniew Bartosiewicz and Ewa Girejko)
Decision Support in Negotiation (Ewa Roszkowska and Tomasz Wachowicz)
Difference and Differential Equations and Their Generalisation on any Time Scales (Ewa Schmeidel and Josef Diblik)
Differential Operators: Algebra, Geometry, and Representations (Antoni Pierzchalski)
Fuzzy Calculus and Its Applications (Witold Kosiński and Dariusz Kacprzak)
Mathematics in Biology and Medicine (Anna Poskrobko)
Quantitative Methods in Economics (Beata Madras-Kobus)
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Differential Operators: Algebra, Geometry, and Representations

Fuzzy Calculus and Its Applications

Mathematics in Biology and Medicine

Quantitative Methods in Economics

Weak Partial Linear and Partial Chain Spaces and Their Geometry

Student Session

Schedule of the Conference

Tuesday, July 1st

10:00–19:30 Registration*
11:00–13:00 Special sessions:
Computer-assisted Formalization of Mathematics I, room 316, 11:00–13:00,
Difference and Differential Equations . . . , room 303, 12:00–13:00
13:00–14:45 Lunch
14:45–15:00 Opening of the conference
15:00–16:00 Plenary session: **Marek Szopa**,
Quantum games and their possible applications in economics
Chair: Ewa Roszkowska, Auditorium (room 301)
16:00–17:00 Plenary session: **Krystyna Kuperberg**,
Algebraic connections between shape theory and dynamical systems
Chair: Artur Korniłowicz, Auditorium (room 301)
17:00–17:30 Coffee break
17:30–19:30 Special sessions:
Algebra I, room 302, 17:30–18:50
Computer Science I, room 304, 17:30–19:00
Computer-assisted Formalization of Mathematics II, room 316, 17:30–18:50
Control Theory and Dynamical Systems I, room 303, 17:30–19:25

*Registration: 10:00–19:30 on July, 1st and at the lunch time on July, 2nd–4th.

Wednesday, July 2nd

09:00–10:00 Plenary session: **Andrzej Szałas**,
Efficient reasoning over partial and inconsistent belief bases
Chair: Magdalena Kacprzak, Auditorium (room 301)
10:00–11:00 Plenary session: **Josef Diblik**,
New criteria for the existence of positive solutions of advanced differential equations
Chair: Ewa Schmeidel, Auditorium (room 301)
11:00–11:30 Coffee break
11:30–13:00 Special sessions:
Algebra II, room 302, 11:30–12:50
Computer Science II, room 206, 11:30–13:00
Computer-assisted Formalization of Mathematics III, room 204, 11:30–13:00
Control Theory and Dynamical Systems II, room 303, 11:30–13:00
Decision Support in Negotiation, room 205, 11:30-13:00
Differential Operators: Algebra, Geometry and Representations, room 304, 11:30–12:55
13:00–15:00 Lunch
15:00–16:00 Plenary session: **Hans Havlicek**,
Preserver problems in geometry
Chair: Krzysztof Prażmowski, Auditorium (room 301)
16:00–16:15 Short break
16:15–17:15 Special sessions:

	Algebra III, room 302, 16:15–17:15
	Computer Science III, room 206, 16:15–17:15
	Computer-assisted Formalization of Mathematics IV, room 204, 16:15–17:15
	Mathematics in Biology and Medicine I, room 303, 16:15–17:35
	Quantitative Methods in Economics I, room 205, 16:15–17:15
	Student Session I, room 316, 16:15–17:15
	Weak Partial Linear and Partial Chain Spaces and Their Geometry I, room 304
17:15–17:45	Coffee break
17:45–19:30	Special sessions:
	Computer Science IV, room 206, 17:45–19:15
	Computer-assisted Formalization of Mathematics V, room 204, 17:45–18:45
	Quantitative Methods in Economics II, room 205, 17:45–19:30
	Student Session II, room 316, 17:45–19:15
	Weak Partial Linear and Partial Chain Spaces and Their Geometry II, room 304

Thursday, July 3rd

09:00–10:00	Plenary session: **Andrzej Skowron**,
	Interactive granular computations and cyber-physical systems,
	Chair: Anna Gomolińska, Auditorium (room 301)
10:00–11:00	Plenary session: **Ulrich Krähmer**,
	Comonads, flat connections, and cyclic homology
	Chair: Ryszard Mazurek, Auditorium (room 301)
11:00–11:30	Coffee break
11:30–12:30	Plenary session: **Bent Ørsted**,
	Universal principles for Pohozaev identities
	Chair: Antoni Pierzchalski, Auditorium (room 301)
12:30–14:00	Special sessions:
	Algebra IV, room 302, 12:30–13:50
	Fuzzy Calculus and Its Applications, room 303, 12:30–14:00
	Student Session III, room 316, 13:30–14:10
14:00–15:45	Lunch
16:00–19:30	Excursion
20:00–...	Conference dinner

Friday, July 4th

09:00–11:00	Special sessions:
	Algebra V, room 302, 09:00–10:40
	Mathematics in Biology and Medicine II, room 303, 09:00–10:40
	Student Session IV, room 316, 10:45–12:05
11:00–11:30	Coffee break
11:30–13:30	Special sessions:
	Student Session IV, room 316, 12:10–13:30
13:30–15:30	Lunch

Closing of the 6PCM

Detailed Program of Special Sessions

Algebra

Tuesday, July 1st, room 302

17:30–18:50 Algebra I, chair: **Piotr Grzeszczuk**

17:30–17:50 **Jan Krempa**, *On finite groups with the basis property*
17:50–18:10 **Agnieszka Stocka**, *On finite matroid groups*
18:10–18:30 **Andre Leroy**, *Continuant polynomials*
18:30–18:50 **Jerzy Matczuk**, *Idempotents and clean elements in ring extensions*

Wednesday, July 2nd, room 302

11:30–12:50 Algebra II, chair: **Andre Leroy**

11:30–11:50 **Witold Tomaszewski,** *On Milnor laws*
11:50–12:10 Leonid F. Barannyk, **Dariusz Klein**, *On finite groups of OTP projective representation type over the ring of integer rational p-adic numbers*
12:10–12:30 **Marcin Skrzyński**, *Remarks on algebraic and geometric properties of the spark of a matrix*
12:30–12:50 Orest Artemovych, **Kamil Kular**, *On the Lie ring of derivations of a semiprime ring*

16:15–17:15 Algebra III, chair: **Jerzy Matczuk**

16:15–16:35 **Małgorzata Jastrzębska**, *On lattices of annihilators in finite dimensional algebras*
16:35–16:55 **Arkadiusz Męcel**, *Conjugacy classes of left ideals of an associative algebra*
16:55–17:15 **Czesław Bagiński**, *On a theorem of elementary number theory and its implications in geometry of closed surfaces*

Thursday, July 3rd, room 302

12:30–13:50 Algebra IV, chair: **Witold Tomaszewski**

12:30–12:50 Ryszard R. Andruszkiewicz, **Magdalena Sobolewska**, *The powerly hereditary property in the lower radical construction*
12:50–13:10 **Izabela Malinowska**, *On generalizations of Dedekind groups*
13:10–13:30 **Małgorzata Hryniewicka**, *Near-rings of quotients*
13:30–13:50 Ryszard R. Andruszkiewicz, **Karol Pryszczepko**, *On the non-torsion almost null rings*

Friday, July 4th, room 302

09:00–10:40 Algebra V, chair: **Czesław Bagiński**

09:00–09:20 Ryszard R. Andruszkiewicz, **Mateusz Woronowicz**, *On TI-groups*
09:20–09:40 **Marzena Filipowicz-Chomko**, Marek Kępczyk, *On $\star(x, y, z)$-ideals*
09:40–10:00 **Kamil Kozłowski**, *On a generalization of Armendariz rings*
10:00–10:20 **Piotr Grzeszczuk**, *Gelfand-Kirillov dimension of algebras with locally nilpotent derivations*
10:20–10:40 **Ryszard Mazurek**, *Rota-Baxter operators on skew generalized power series rings*

Computer Science

Tuesday, July 1st, room 304

17:30–19:00 Computer Science I, chair: **Agnieszka Dardzińska-Głębocka**

17:30–18:00 **Anna Gomolińska**, Marcin Wolski,
The standard rough inclusion function as a basis for similarity indices

18:00–18:30 **Bożena Woźna-Szcześniak**,
On the SAT-based verification of communicative commitments

18:30–19:00 **Mariusz Rybnik**, Władysław Homenda, *Similarity-based searching in structured spaces of music information*

Wednesday, July 2nd, room 206

11:30–13:00 Computer Science II, chair: **Andrzej Skowron**

11:30–12:00 **Jolanta Koszelew**,
Optimization problems and methods applicable in intelligent tourist travel planners

12:00–12:30 **Joanna Karbowska-Chilińska**, Paweł Zabielski, *Combining genetic algorithm and path relinking for solving Orienteering Problem with Time Windows*

12:30–13:00 Magdalena Kacprzak, **Bartłomiej Starosta**,
An approach to making decisions with metasets

16:15–17:15 Computer Science III, chair: **Bożena Woźna-Szcześniak**

16:15–16:45 **Agnieszka Dardzińska-Głębocka**, Anna Łobaczuk-Sitnik,
Action rules mining in laryngological diseases

16:45–17:15 **Zenon Sadowski**, *Do there exist complete sets for promise classes?*

17:45–19:15 Computer Science IV, chair: **Andrzej Szałas**

17:45–18:15 **Magdalena** Kacprzak, **Anna** Sawicka, *Dialogue as a game*

18:15–18:45 **Katarzyna Woronowicz**, *Multi-valued De Morgan gate*

18:45–19:15 **Krzysztof Ostrowski**, *Comparison of different graph weights representations used to solve the time-dependent Orienteering Problem*

Decision Support in Negotiation

Wednesday, July 2nd, 11:30–13:00, room 205

Chair: **Krzysztof Piasecki**

11:30–11:50 **Ewa Roszkowska**, Tomasz Wachowicz, *How do the decision makers decide? A multiple criteria decision making experiment*

11:50–12:10 **Dorota Górecka**, Ewa Roszkowska, Tomasz Wachowicz, *MARS approach in scoring negotiation offers for the verbally defined preferences of the negotiators*

12:10–12:30 **Marek Szopa**, *How quantum games can support negotiation?*

12:30–13:00 *Discussion*

Computer-assisted Formalization of Mathematics

– In memoriam of Andrzej Trybulec

Tuesday, July 1st, room 316

11:00–13:00 Computer-assisted Formalization of Mathematics I, chair: **Artur Korniłowicz**

11:00–11:10 *Opening*
11:10–12:00 **John Harrison,** *Formal proofs of hypergeometric sums*
12:00–12:20 **Marco B. Caminati**, Manfred Kerber, Colin Rowat,
Budget imbalance criteria for auctions: a formalized theorem
12:20–12:40 **Kazuhisa Nakasho**, Hiroyuki Okazaki, Hiroshi Yamazaki, Yasunari Shidama,
Formalization of fundamental theorem of finite abelian groups in Mizar
12:40–13:00 **Grzegorz Bancerek**, *Feasible analysis of algorithms with Mizar*

17:30–18:50 Computer-assisted Formalization of Mathematics II, chair: **Adam Naumowicz**

17:30–17:50 **Adam Grabowski**, *Formal characterization of almost distributive lattices*
17:50–18:10 **Maciej Goliński**,
Formalizing cryptographic algorithms in Mizar with the Enigma example
18:10–18:30 Adam Grabowski, **Christoph Schwarzweller**,
Towards standard environments for formalizing mathematics
18:30–18:50 **Karol Pąk**, *Improving legibility of proof scripts based on quantity of introduced labels*

Wednesday, July 2nd, room 204

11:30–13:00 Computer-assisted Formalization of Mathematics III, chair: **Josef Urban**

11:30–12:20 **Geoff Sutcliffe**,
The TPTP typed first-order form with arithmetic: the language and some applications
12:20–12:40 **Robert Y. Lewis**, Jeremy Avigad, Cody Roux, *A heuristic prover for real inequalities*
12:40–13:00 **Mario Carneiro**, *Natural deduction in the Metamath proof language*

16:15–17:15 Computer-assisted Formalization of Mathematics IV, chair: **Karol Pąk**

16:15–16:35 Alexander Lyaletski, **Mykola Nikitchenko**,
Logical and semantic investigations in Kyiv school of automated reasoning
16:35–16:55 **Anna Zalewska**, *On an alternative approach to Skolemising*
16:55–17:15 **Radosław Piliszek**, *Implementing a parser for the WS-Mizar language*

17:45–18:45 Computer-assisted Formalization of Mathematics V, chair: **Adam Grabowski**

17:45–18:05 **Josef Urban**, *Parallelizing Mizar*
18:05–18:25 **Adam Naumowicz**, *Combining Mizar with Logic2CNF SAT Solver*
18:25–18:45 **Artur Korniłowicz**, *Equalities in Mizar*

Control Theory and Dynamical Systems

Tuesday, July 1st, room 303

17:30–19:25 Control Theory and Dynamical Systems I, chair: **Zbigniew Bartosiewicz**

17:30–18:10 Keynote: **Daniel Franco**, *Getting rid of chaotic population dynamics*
18:10–18:30 Zbigniew Bartosiewicz, Ulle Kotta, Branislav Rehak, Maris Tonso, **Małgorzata Wyrwas**, *Polynomial accessibility condition of nonlinear control systems on homogeneous time scales*
18:30–18:50 **Ewa Girejko**, Dorota Mozyrska, Małgorzata Wyrwas, *A necessary condition of viability for fractional equations with the Caputo derivative*
18:50–19:10 **Zbigniew Zaczkiewicz**, *Relative observability, duality for fractional differential algebraic delay systems with jumps*
19:10–19:25 **Monika Ciulkin**, *A classification of linear controllable time-varying systems on time scales*

Wednesday, July 2nd, room 303

11:30–13:00 Control Theory and Dynamical Systems II, chair: **Ewa Girejko**

11:30–11:50 **Piotr Mormul**, *Nilpotency issues in the car+trailers' systems*
11:50–12:05 **Dorota Mozyrska**, Małgorzata Wyrwas, *Why fractional systems are not dynamical systems?*
12:05–12:25 **Zdenek Svoboda**, *Asymptotic properties of delayed matrix functions*
12:25–12:45 Keynote: **Andreas Ruffing**, *Quantum difference-differential equations*
12:45–13:00 **Zbigniew Bartosiewicz**, *Global observability of nonlinear systems*

Difference and Differential Equations and Their Generalisation on any Time Scales

Tuesday, July 1st, 12:00-13:00, room 303

Chair: **Ewa Schmeidel**

12:00–12:15 **Ewa Schmeidel**, *Boundedness and stability of discrete Volterra equations*
12:15–12:30 Jarosław Mikołajski, Ewa Schmeidel, **Joanna Zonenberg**, *Comparison of boundedness of solutions of differential and recurrence equations*
12:30–12:45 **Robert Jankowski**, Ewa Schmeidel, Joanna Zonenberg, *Boundedness of solutions of neutral type nonlinear difference system with deviating arguments*
12:45–13:00 **Agata Bezubik**, Severin Pošta, *On discretization of polynomials corresponding to symmetric and antisymmetric functions in four variables*

Differential Operators: Algebra, Geometry and Representations

Wednesday, July 2nd, 11:30-12:55, room 304

Chair: **Antoni Pierzchalski**

11:30–11:55 **Anna Kimaczyńska**, *Natural differential operators in the symmetric bundle*
12:00–12:25 **Bogdan Balcerzak**, *Dirac operators on Lie algebroids*
12:30–12:55 **Agnieszka Najberg**, *Natural differential operators on symplectic manifolds*

Fuzzy Calculus and Its Applications

Thursday, July 3rd, 12:30–14:00, room 303

Chair: **Ewa Roszkowska**

12:30–12:50 **Takashi Mitsuishi**, Koji Saigusa, Yasunari Shidama, *Periodic fuzzy set on circular coordinates*
12:50–13:10 Irena Sobol, **Dariusz Kacprzak**, *Optimizing inventory of a firm under fuzzy data*
13:10–13:30 **Dariusz Kacprzak**, Ewa Roszkowska, *The application of ordered fuzzy numbers in the SAW procedure*
13:30–14:00 *Discussion*

Mathematics in Biology and Medicine

Wednesday, July 2nd, room 303

16:15–17:35 Mathematics in Biology and Medicine I, chair: **Anna Poskrobko**

16:15–16:55 Keynote: **Priti Kumar Roy**, in collaboration with Shubhankar Saha, Amar Nath Chatterjee, Sonia Chowdhury, *Effect of awareness programs with HAART in controlling the disease HIV/AIDS: A mathematical approach*
16:55–17:15 **Sumit Nandi**, Dibyendu Biswas, Amar Nath Chatterjee, Priti Kumar Roy, *Effect of delay during transmission of the disease Cutaneous Leishmaniasis: A mathematical approach*
17:15–17:35 **Marzena Filipowicz-Chomko**, Ewa Girejko, Anna Poskrobko, *Criterions of multidimensional ν-similarity for birth-death processes*

Friday, July 4th, room 303

09:00–10:40 Mathematics in Biology and Medicine II, chair: **Anna Poskrobko**

09:00–09:40 Keynote: **Daniel Franco**, *Effects of environmental protection in population dynamics*
09:40–10:00 **Antoni Leon Dawidowicz**, Anna Poskrobko, *Stability problem for the age-dependent predator-prey model*
10:00–10:20 **Ewa Girejko**, Anna Poskrobko, *Families of ν-similar birth-death processes and limiting conditional distributions*
10:20–10:40 **Anna Poskrobko**, Antoni Leon Dawidowicz, *Asymptotic properties of the von Foerster-Lasota equation and indices of Orlicz space*

Quantitative Methods in Economics

Wednesday, July 2nd, room 205

16:15–17:15 Quantitative Methods in Economics I, chair: **Ewa Roszkowska**

16:15–17:00 KEYNOTE: **Krzysztof Piasecki**, *Intuitionistic present value – an axiomatic approach*
17:00–17:15 *Discussion*

17:45–19:30 Quantitative Methods in Economics II, chair: **Ewa Roszkowska**

17:45–18:05 **Dorota Kozioł-Kaczorek**, *The use of combined multicriteria method for the valuation of real estate*
18:05–18:25 Paweł Jamróz, **Grzegorz Koronkiewicz**, *Comparison of the tails of market return distributions*
18:25–18:45 **Paweł Konopka**, *Financing startups business – grants or preferential loans?*
18:45–19:05 **Kamil Bienias**, Tomasz Czyżycki, *Applications of line and surface integrals to economic issues*
19.05–19.30 *Discussion*

Weak Partial Linear and Partial Chain Spaces and Their Geometry

Wednesday, July 2nd, room 304

16:15–17:10 Weak Partial Linear and Partial Chain Spaces and Their Geometry I
chair: **Krzysztof Prażmowski**

16:15–16:40 **Andrzej Matraś**, *On some configuration on finite non-desarguesian projective planes*
16:45–17:10 **Małgorzata Prażmowska**, *Projective realizability of Veronese spaces*

17:45–19:30 Weak Partial Linear and Partial Chain Spaces and Their Geometry II
chair: **Andrzej Matraś**

17:45–18:10 **Edyta Bartnicka**, *The projective line over finite associative ring with unity*
18:10–18:35 **Krzysztof Prażmowski**, *Weak partial chain spaces and their products*
18:40–19:05 **Krzysztof Petelczyc**, *On affinization of Segre products of partial linear spaces*
19:05–19:30 **Mariusz Żynel**, *Affinization of Segre products of Grassmann spaces embeddable into projective spaces: Applications and examples*

Student Session

Wednesday, July 2nd, room 316

16:15–17:15 Student Session I, chair: **Ryszard Mazurek**

16:15–16:35 Jarosław Woźniak, **Mateusz Firkowski**, *Stability of a partially damped rotating Timoshenko beam*
16:35–16:55 **Magdalena Bielawska**, *How to create effective admissions system?*
16:55–17:15 **Hubert Anisimowicz**, *Circulant matrices and their application to solving polynomial equations*

17:45–19:15 Student Session II, chair: **Ryszard Mazurek**

17:45–19:15 KEYNOTE: **Marek Drużdżel**, *Being an academic researcher: What's it like?* (in Polish)

Thursday, July 3rd, room 316

13:30–14:10 Student Session III, chair: **Ewa Girejko**

13:30–13:50 **Daniel Strzelecki**, *Conley index and its applications*
13:50–14:10 **Kamil Kozłowski**, *Around Banach-Tarski paradox*

Friday, July 4th, room 316

10:45–13:30 Student Session IV, chair: **Marzena Filipowicz-Chomko**

10:45–11:05 **Jakub Kabat**, *Riesz spaces, Riesz homomorphisms, and the Banach-Stone theorem*
11:05–11:25 **Adam Soroczyński**, *Distributive rings and modules*
11:25–11:45 **Adam Tucholski**, *The axiom of choice in algebra*
11:45–12:05 **Kamil Szymon Jadeszko**, *The best card trick – power of permutations*
12:05–12:10 Short break
12:10–12:30 **Paweł Olszewski**, *Algebraic proof of the Fundamental Theorem of Algebra*
12:30–12:50 **Katarzyna Ignatiuk**, *The time scales calculus*
12:50–13:10 **Marcin Żukowski**, *Tomographic image reconstruction methods*
13:10–13:30 **Krystian Moraś**, *Curves and splines in computer graphics*

Plenary Lectures

Quantum games and their possible applications in economics

Marek Szopa

University of Silesia, Katowice, Poland

In this lecture we define the notion of a quantum game, for which players' strategies are parameterized by 3D rotations. Meta-strategies of the quantum game are correlated through the mechanism of quantum entanglement and the result of the game is obtained by the collapse of the wave function. They can be realized by quantum computers and they are completely safe against eavesdropping. Classical games are particular examples of quantum games. Quantum games have richer class of Nash equilibria than classical games. In this presentation we show examples of quantum games and their corresponding classical counterparts and we show how one can utilize the additional quantum meta-strategies.

Algebraic connections between shape theory and dynamical systems

Krystyna Kuperberg

Auburn University, Alabama, USA

The flow box structure of topological dynamical systems defined on manifolds relates closely to isotopies and the topological methods used by Ważewski to prove the retract theorem and to study the asymptotic behavior of trajectories. A finite dimensional attractor has the shape of a polyhedron and other stability theory objects bear strong similarity to UV-sets in shape theory.

This lecture addresses the application of Borsuk's shape theory and Vietoris-Čech homology and cohomology to continuous dynamics, in particular to the special case of differentiable flows.

Efficient reasoning over partial and inconsistent belief bases

Andrzej Szałas

University of Warsaw, Poland
University of Linköping, Sweden

Many fundamental issues in reasoning methods designed for intelligent systems manifest themselves, among others, when such systems are situated in dynamic and unpredictable environments. Typically, processes of information acquisition and fusion inevitably lead to incomplete and frequently inconsistent beliefs. Traditional logical approaches are not well suited both to specifying models and to reasoning in such circumstances. Partial and/or inconsistent information leads to uncertain conclusions while inconsistencies make traditional logics explosive by allowing one to derive arbitrary, unsupported conclusions.

The lecture will be devoted to a computationally friendly framework addressing the problems of partial and inconsistent information. Surprisingly enough, a couple of relatively simple constructs, combined together, appear sufficient to model heterogeneous information sources as well as to express advanced reasoning schemes handling partial, uncertain and inconsistent information. In particular, lightweight versions of a variety of reasoning methods, including nonmonotonic and defeasible ones, can be expressed and combined arbitrarily in a well-understood and tractable manner. The key idea depends on structuring knowledge and applying carefully designed logical calculus.

New criteria for the existence of positive solutions of advanced differential equations

Josef Diblik

Brno University of Technology, Czech Republic

The monotone iterative technique is used to get new positivity criteria to differential equations with advanced arguments. The attention will be focused on linear differential equations as well. New explicit criteria for the existence of a positive solution will be presented. An overview of known relevant criteria will be given together with relevant comparisons.

Keywords: positive solution, advanced differential equation, monotone iterative technique

Preserver problems in geometry

Hans Havlicek

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The study of certain transformation groups on various spaces of real and complex matrices (rectangular, symmetric, Hermitian, alternating) was initiated by L. K. Hua around the year 1945. The aim of his study was to characterise those groups by as few geometric invariants as possible.

An essential notion in Hua's work is that of *adjacent* (or, in a different terminology, *coherent*) matrices: One says that matrices A, B from a matrix space are adjacent if $A - B$ has rank 1, except for alternating matrices, where the rank of $A - B$ is assumed to be 2. It is worth noting that Hua's early papers include also extra assumptions, like continuity. However, all of them soon turned out to be superfluous. Later, the results of Hua were extended in various directions by going over to arbitrary commutative or even non-commutative ground fields. Furthermore, so called *projective matrix spaces* allowed to incorporate the work of W. L. Chow (1949) about adjacency preserving transformations of Grassmannians and certain subsets of Grassmannians, like the set of totally isotropic subspaces of a symplectic form. Here two subspaces (of the same dimension, say m) are called *adjacent* if their intersection has dimension $m - 1$. The groups under consideration are certain classical groups, like the group of semilinear bijections of the vector space underlying the Grassmannian.

In the work of Hua and Chow only *bijective* mappings that preserve adjacency *in both directions* were exhibited, and a series of beautiful (and powerful) characterisations were obtained. The two last decades have witnessed a tremendous advance: Much of the original results may be established under even weaker assumptions. A first step was taken by exhibiting bijective mappings that preserve

adjacency *in one direction* only. Later *surjective* rather than bijective mappings were considered, and also other assumptions were successfully relaxed. However, what might be an *optimal version* of various results is still quite unclear. Another development was to replace the relation of adjacency by some other geometric notion.

The aim of this lecture is to present an overview of several old and recent findings about these and related *preserver problems*, but – due to the lack of time – we plan to put a focus on transformations of Grassmannians.

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Interactive granular computations and cyber-physical systems

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Cyber-Physical Systems (**CPS**) will help us to interact with the physical world just as the Internet helps us to interact with one another. It is predicted that applications based on CPS will have enormous societal impact and will bring enormous economic benefit. We discuss computational model for CPS based on interactive complex granules (c-granules, for short). C-granules are controlled by agents. Our approach is based on the **Wisdom Technology** meta-equation

$$\mathbf{wisdom} = \mathbf{interactions} + \mathbf{adaptive\_judgment} + \mathbf{knowledge\_bases}.$$

The understanding of interactions is the critical issue of CPS treated as complex systems. Using c-granules one can model interactions of agents with the physical world and represent perception of interactions in the physical world by agents. Adaptive judgment allows agents to reason about c-granules and interactive computations performed on them. In adaptive judgment, different kinds of reasoning are involved such as deduction, induction, abduction or reasoning by analogy. In the approach, an important role is also played by knowledge bases and interactions of agents with them. Some illustrative applications of the proposed approach related to real-life projects (e.g., respiratory failure, UAV control, algorithmic trading, sunspot classification, semantic search engine, firefighter safety) are reported. We emphasize the pivotal role of the proposed approach for risk management in CPS.

Comonads, flat connections, and cyclic homology

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Cyclic homology is a homology theory for algebras. For commutative algebras it coincides with the De Rham cohomology of the corresponding affine scheme. In the noncommutative case, it is used to develop an analogue of characteristic classes of vector bundles defined on K-theory. These classes are the key ingredients in the generalisations of the Atiyah-Singer index theorem due to Connes, Moscovici, and others.

In this talk I will give a historic overview of this theory and then discuss the still open question how cyclic homology fits into the general theory of homological algebra, discussing recent and new results, e.g. of Kaledin, of Boehm and Srefan, and of Kowalig and myself.

Universal principles for Pohozaev identities

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In this talk we shall study certain invariants on Riemannian manifolds, such as curvature invariants and their interplay with conformal vector fields. An example is the Kazdan-Warner identity, which gives a necessary condition for a function to be a scalar curvature function.

In joint work with R. Gover we have found several such identities, also including the Pohozaev identity by using a variant of Noether's theorem.

Session: Algebra

On the non-torsion almost null rings

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This talk is devoted to the study of some subclasses of H-rings i.e. rings in which every subring is an ideal. In the description of H-rings a central role play, so-called almost null rings. In this talk we present and classify, up to an isomorphis, some general examples of non-torsion almost null rings.

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The powerly hereditary property in the lower radical construction

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All rings in this talk are associative but are not required to have a unity or to be commutative.

A radical $\mathcal{S}$ is said to be *left strong* if for every left ideal A of R if $A \in \mathcal{S}$, then the ideal of R generated by A is in $\mathcal{S}$. It is well known (see [4]) that for every homomorphically closed class $\mathcal{M}$, there exists a least radical class $l(\mathcal{M})$, which contains $\mathcal{M}$ and there exists a least left strong radical $ls(\mathcal{M})$, which contains $\mathcal{M}$.

The lower radical (resp. the lower left strong radical) determined by a class $\mathcal{M}$ can be described by the sum of Kurosh's chain $\{\mathcal{M}_\alpha\}$ (resp. left Kurosh's chain $\{\mathcal{M}^\alpha\}$). Namely, define $\mathcal{M}_1 = \mathcal{M}^1 = \mathcal{M}$ and for $\alpha > 1$ define $\mathcal{M}_\alpha$ (resp. $\mathcal{M}^\alpha$) to be the class of all rings R such that every nonzero homomorphic image of R contains a nonzero ideal (resp. left ideal) belonging to $\mathcal{M}_\beta$ (resp. $\mathcal{M}^\beta$) for some $\beta < \alpha$.

The question of the stabilization of Kurosh's chains has been investigated in a number of papers (see [1, 2, 1, 2, 7–9]). In [2] it was show that if $\mathcal{M}$ is a hereditary and homomorphically closed class of rings, then $l(\mathcal{M}) = \mathcal{M}_3$. From [8] it follows that $l(\mathcal{M}) = \mathcal{M}_3$ if $\mathcal{M}$ is a homomorphically closed class of nilpotent rings. A common generalization of these classes are powerly hereditary classes. Namely, a homomorphically closed class $\mathcal{M}$ of assocciative rings is called *powerly hereditary* if for every ring $R \in \mathcal{M}$ there is an integer $n > 1$ such that $R^n \in \mathcal{M}$.

We prove that Kurosh's chain determined by a powerly hereditary class stabilizes at step 3. Under some additional assumptions, left Kurosh's chain stabilizes at step 4.

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On TI-groups

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An abelian group $(A, +, 0)$ is called a TI-group if every associative ring with additive group A is filial. We present results concerning the structure of TI-groups. In the talk, the structure theorem describing torsion TI-groups will be proved and the structure of the torsion part of mixed TI-groups will be described. Furthermore, we prove that every abelian torsion-free group of rank one is a TI-group. Numerous examples of TI-groups will be given.

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On the Lie ring of derivations of a semiprime ring

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Let $\mathrm{Der}(R)$ be the Lie ring of derivations of an associative ring R (possibly without identity). In the talk, we will present an elementary proof of the following fact.

Theorem 1. *If R is semiprime, then either $\mathrm{Der}(R) = \{0\}$, or $\mathrm{Der}(R)$ is not nilpotent.*

The talk will be based mainly on [1].

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On a theorem of elementary number theory and its implications in geometry of closed surfaces

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Let Λ be a Fuchsian group with the presentation

$$\langle x_1, x_2, x_3 \mid x_1^{m_1} = x_2^{m_2} = x_3^{m_3} = x_1x_2x_3 = 1\rangle \tag{1}$$

For a smooth epimorphism $\theta : \Lambda \to \langle a | \; a^N = 1\rangle$ we have

$$\theta(x_1) = a^{mN/m_1},\ \theta(x_2) = a^{kN/m_2},\ \theta(x_1) = a^{lN/m_3},$$

where $\gcd(m, m_1) = \gcd(k, m_2) = \gcd(l, m_3) = 1$ and

$$m\left(\frac{N}{m_1}\right) + k\left(\frac{N}{m_2}\right) + l\left(\frac{N}{m_3}\right) \equiv 0 \pmod N.$$

Let m' be the inversion of m modulo m_1. Let also A_1' be the maximal divisor of $A_1 = N/m_1$ coprime to m_1. Then by the Chinese Remainder Theorem there exists α such that $\alpha \equiv m' \pmod{m_1}$ and $\alpha \equiv 1 \pmod{A_1'}$. Then $\gcd(\alpha, N) = 1$ and

$$(a^{mN/m_1})^\alpha = (a^{N/m_1})^{m\alpha} = a^{N/m_1}$$

and so we see that, up to a powering of fixed generator of $\langle a | \; a^N = 1\rangle$, our θ is defined by

$$\theta(x_1) = a^{N/m_1},\ \theta(x_2) = a^{kN/m_2},\ \theta(x_1) = a^{lN/m_3}, \tag{2}$$

where $\gcd(k, m_2) = \gcd(l, m_3) = 1$ and

$$\frac{N}{m_1} + k\left(\frac{N}{m_2}\right) + l\left(\frac{N}{m_3}\right) \equiv 0 \pmod N.$$

Let $\mathcal{K} = \{k < m_2 \,|\, \gcd(k, m_2) = 1\}$. Then $K = \varphi(m_2)$ is its cardinality, where φ is the Euler function. Let also $\mathcal{L}$ be the subset of $\mathcal{K}$ consisting of those k for which

$$a^{N/m_1+kN/m_2} \tag{3}$$

has order m_3 and let L be its cardinality. We see that with these notation we have

Lemma 1. *There are just L surface-kernel epimorphisms $\theta : \Lambda \to \langle a | \; a^N = 1\rangle$, where Λ is a Fuchsian group with the presentation* (1) for which $\theta(x_1) = a^{N/m_1}$. □

Now let $\mathcal{S}$ be the stabilizer of a^{N/m_1} in $\operatorname{Aut}\langle a\rangle = \mathbb{Z}_N^*$. Then for its cardinality S we have

Lemma 2. $S = \varphi(N)/\varphi(m_1)$.

Lemma 3. *Each element of $\mathcal{S}$ acts on $\mathcal{K}$ without fixed points and so in particular the group $\mathcal{S}$ acts faithfully on $\mathcal{K}$.*

[⋆] The talk is based on the manuscript: Cz. Bagiński, M. Carvacho, G. Gromadzki, R. Hidalgo, *On periodic self-homeomorphisms of closed orientable surfaces determined by their orders*, submitted.

For counting L it will be crucial the following decomposition.

$$\begin{cases} m_1 = AA_2A_3 \\ m_2 = AA_1A_3 \\ m_3 = AA_1A_2 \end{cases} \tag{4}$$

where $A = \gcd(m_1, m_2, m_3)$, $A_k = \gcd(m_i/A, m_j/A)$, for $k \neq i, j$. For, we have

$$N = \operatorname{lcm}(m_1, m_2, m_3) = \operatorname{lcm}(m_1, m_2) = \operatorname{lcm}(m_1, m_3) = \operatorname{lcm}(m_2, m_3),$$

where $N = AA_1A_2A_3$, $N/m_i = A_i$. Furthermore we know that if N is even, then the number of those A_i which are odd is even and therefore, since all A_i can not be even, only one of them is even. With these notations, elements (3) become

$$a^{A_1+kA_2}$$

and so the set $\mathcal{L}$ becomes

$$\mathcal{L} = \{k < AA_1A_3 \mid \gcd(A_1 + kA_2, N) = A_3,\ \gcd(k, AA_1A_3) = 1\}. \tag{5}$$

We define $\psi(1) = 1$ and given a decomposition $n = p_1^{\alpha_1} \dots p_r^{\alpha_r} > 1$

$$\psi(n) = \prod_{i=1}^{r} (p_i - 2)p_i^{\alpha_i - 1}. \tag{6}$$

With this definition we have

Theorem 1 *Let C be the biggest divisor of A coprime with $A_1A_2A_3$ and let $B = A/C$. Then*

$$L = \varphi(A_1B)\psi(C). \tag{7}$$

Theorem 2 *Let C be the biggest divisor of A coprime with $A_1A_2A_3$ and let $B = A/C$. Then $L = S$ if and only if $B \in \{1, 2\}$ and $C \in \{1, 3\}$.*

On finite groups of OTP projective representation type over the ring of integer rational p-adic numbers

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In this paper we continue the study as begun in [1] – [1].

Let $\mathbb{Z}_p$ be the ring of integer rational p-adic numbers, $\mathbb{Z}_p^*$ the unit group of $\mathbb{Z}_p$, K the residue class field of $\mathbb{Z}_p$ by (p), and $G = G_p \times B$ a finite group, where G_p is a p-group and B is a p'-group. Denote by $\mathbb{Z}_p^\lambda G$ the twisted group algebra of G over $\mathbb{Z}_p$ with a 2-cocycle $\lambda \in Z^2(G, \mathbb{Z}_p^*)$. Every cocycle $\lambda \in Z^2(G, \mathbb{Z}_p^*)$ is cohomologous to $\mu \times \nu$, where μ is the restriction of λ to $G_p \times G_p$ and ν is the restriction of λ to $B \times B$. We assume that every cocycle $\lambda \in Z^2(G, \mathbb{Z}_p^*)$ under consideration satisfies the condition $\lambda = \mu \times \nu$, and all $\mathbb{Z}_p^\lambda G$-modules are assumed to be finitely generated left $\mathbb{Z}_p^\lambda G$-modules which are $\mathbb{Z}_p$-free. The algebra $\mathbb{Z}_p^\lambda G$ is defined to be of OTP representation type if every indecomposable $\mathbb{Z}_p^\lambda G$-module is isomorphic to the outer tensor product $V\#W$, where V is an indecomposable $\mathbb{Z}_p^\mu G_p$-module and W is an irreducible $\mathbb{Z}_p^\nu B$-module. The group $G = G_p \times B$ is defined to be of OTP projective $\mathbb{Z}_p$-representation type if there exists a cocycle $\lambda \in Z^2(G, \mathbb{Z}_p^*)$ such that the algebra $\mathbb{Z}_p^\lambda G$ is of OTP representation type. The group $G = G_p \times B$ is said to be of purely OTP projective $\mathbb{Z}_p$-representation type if $\mathbb{Z}_p^\lambda G$ is of OTP representation type for any $\lambda \in Z^2(G, \mathbb{Z}_p^*)$.

We prove the following theorems.

Theorem 1. *Let $G = G_p \times B$, where G_p is abelian and $p \neq 2$. Assume also that G_p is not of type (p^n, p). The group G is of OTP projective $\mathbb{Z}_p$-representation type if and only if one of the two following conditions is satisfied:*

(i) G_p is cyclic.

(ii) K is a splitting field for $K^\sigma B$ for every $\sigma \in Z^2(B, K^*)$.

Theorem 2. *The group $G = G_p \times B$ is of purely OTP projective $\mathbb{Z}_p$-representation type if and only if one of the following three conditions is satisfied:*

(i) G_p is a cyclic group of order p^r, $r \leq 2$;

(ii) $p \neq 2$ and there exists a finite central group extension $1 \to A \to \hat{B} \to B \to 1$ such that any projective K-representation of B lifts projectively to an ordinary K-representation of $\hat{B}$ and K is a splitting field for $\hat{B}$.

(iii) $p = 2$ and K is a splitting field for B.

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On $\star(x, y, z)$-ideals

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The concept of $*$-ideals was introduced by Puczyłowski in [3]. According to it, I is a $*$*-ideal* of a ring A if I is an ideal of any ring B containing A as an ideal. Generalizing this, we give in [1] a new definition of $*(x, y, z)$-ideal, where x, y and z are elements from the set $\{t, l, r\}$, for $A \lhd_t B$, $A \lhd_l B$ and $A \lhd_r B$ being an ideal, a left ideal and a right ideal of a ring B, as follows: *A subring L of a ring A which is $L \lhd_x A$ will be called a $*(x, y, z)$-ideal if for every ring B such that $A \lhd_y B$ also $L \lhd_z B$.*

In this talk, the characterization of $*(x, y, y)$-ideals and $*(x, t, y)$-ideals, for $t \neq y$ will be presented. We will also describe rings not containing nontrivial $*(x, y, z)$-ideals (i.e. $*(x, y, z)$-simple rings) in almost all cases considered above (except the $\star(x, y, y)$-simple rings, where $y \in \{l, r\}$ and $x \neq y$). The description extends the results obtained in [2] and [3].

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Gelfand-Kirillov dimension of algebras with locally nilpotent derivations

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Joint work with Jeffrey Bergen (DePaul University, Chicago, USA)

Let R be an algebra over a field F of characteristic 0 and we let $\mathrm{GKdim}(R)$ denote the Gelfand-Kirillov dimension of R as an algebra over F. An F-linear function $\delta\colon R \to R$ is a derivation if

$$\delta(rs) = \delta(r)s + r\delta(s)$$

for all $r, s \in R$. We let

$$R^\delta = \{r \in R \mid \delta(r) = 0\},$$

denote the ring of invariants of R under δ. We say that δ is locally nilpotent if, for every $r \in R$, there exists $n = n(r) \geq 1$ such that $\delta^n(r) = 0$.

We will discuss recent results ([4]) on the structure of algebras of finite GK dimension and having locally nilpotent derivations. This work is motivated by the result of Bell and Smoktunowicz [2], where it is shown that if R is a finitely generated domain over an algebraically closed field of characteristic 0 with a locally nilpotent derivation $\delta \neq 0$ such that the GK dimension of R is in the interval $[2, 3)$, then R has GK dimension 2. In [2], the invariants R^δ have GK dimension 1 and are commutative. Therefore, the result in [2] can be viewed as saying that if $\mathrm{GKdim}(R) < \mathrm{GKdim}(R^\delta) + 2$ and R^δ has some special properties, then $\mathrm{GKdim}(R) = \mathrm{GKdim}(R^\delta) + 1$. We obtain results of this type in more general situations.

The following result makes use of work in [1] and [3].

Theorem 3 *Let R be an algebra over a field of characteristic 0 with a locally nilpotent derivation δ. If R^δ is prime and Goldie then*

1. *R is prime and Goldie.*
2. *The set T of all regular elements of R^δ is a left Ore set in R.*
3. *There exists $x \in T^{-1}R$ such that $\delta(x) = 1$ and a derivation $d\colon Q(R^\delta) \to Q(R^\delta)$ such that*

$$T^{-1}R = Q(R^\delta)[x; d].$$

4. *If R^δ is prime and satisfies a polynomial identity, then there exists $y \in R$ such that $\delta(y) \in Z(R^\delta)$ and a derivation $d_y\colon Q(R^\delta) \to Q(R^\delta)$ such that*

$$T^{-1}R = Q(R^\delta)[y; d_y].$$

Corollary 4 *Let R be an algebra over a field of characteristic 0 having a locally nilpotent derivation $\delta \neq 0$ such that R^δ is prime and satisfies a polynomial identity. Then there is a derivation d of R^δ, induced by some $x \in R$, such that $R^\delta[x;d] \subseteq R \subseteq Q(R^\delta)[x;d]$.*

Theorem 5 *Let R be a finitely generated algebra over a field of characteristic 0 having a locally nilpotent derivation $\delta \neq 0$. If $\mathrm{GKdim}(R) < \mathrm{GKdim}(R^\delta) + 2 < \infty$ where R^δ is prime and satisfies a polynomial identity, then either*

1. *$R \subseteq Q(R^\delta)[x]$, or*
2. *$R \subseteq B[x;d]$, where d is a derivation of R^δ and B is a finitely generated d-stable subalgebra of $Q(R^\delta)$.*

Theorem 6 *Let R be a finitely generated algebra over a field of characteristic 0 having a locally nilpotent derivation $\delta \neq 0$. If $\mathrm{GKdim}(R) < \mathrm{GKdim}(R^\delta) + 2 < \infty$, where R^δ is prime and satisfies a polynomial identity, then $\mathrm{GKdim}(R) = \mathrm{GKdim}(R^\delta) + 1$. In particular, $\mathrm{GKdim}(R)$ must be a positive integer.*

When R is a domain, we can remove the assumption that R is finitely generated.

Corollary 7 *Let R be a domain over a field of characteristic 0 having a locally nilpotent derivation $\delta \neq 0$. If $\mathrm{GKdim}(R) < \mathrm{GKdim}(R^\delta) + 2 < \infty$, where R^δ satisfies a polynomial identity, then $\mathrm{GKdim}(R) = \mathrm{GKdim}(R^\delta) + 1$. In particular, $\mathrm{GKdim}(R)$ must be a positive integer.*

Corollary 7 has an immediate application to skew polynomial rings over domains which satisfy a polynomial identity.

Corollary 8 *Let R be a domain over a field of characteristic 0 which satisfies a polynomial identity. If d is a derivation of R such that $\mathrm{GKdim}(R[x;d]) < \mathrm{GKdim}(R) + 2 < \infty$, then $\mathrm{GKdim}(R[x;d]) = \mathrm{GKdim}(R) + 1$. In particular, $\mathrm{GKdim}(R[x;d])$ must be a positive integer.*

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Near-rings of quotients

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Near-rings satisfy all the axioms satisfied by rings except commutativity of addition and one of the distributivity laws. For convenience we require the right distributivity law to be satisfied. Near-rings arise naturally when the set of all maps of an (additively written) group into itself is given by two operations, addition defined pointwise and multiplication as the usual composition of maps. In the talk we concentrate on the problem of the existence of right (respectively, left) near-rings of quotients.

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On lattices of annihilators in finite dimensional algebras

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In this talk $\mathbb{K}$ will be a field and A an associative $\mathbb{K}$-algebra with $1 \neq 0$.

It is well know that the set $\mathcal{I}_l(A)$ of all left ideals and the set $\mathcal{I}_r(A)$ of all right ideals in A are complete modular lattices.

Further, if $X \subseteq A$ is a subset then let $\mathrm{L}_A(X) = \mathrm{L}(X)$ be the left annihilator of X in A and let $\mathrm{R}_A(X) = \mathrm{R}(X)$ be the right annihilator of X in A. Let $\mathcal{A}_l(A)$ be the set of all left annihilators in A and let $\mathcal{A}_r(A)$ be the set of all right annihilators in A. Then $\mathcal{A}_l(A) \subseteq \mathcal{I}_l(A)$ and $\mathcal{A}_r(A) \subseteq \mathcal{I}_r(A)$ are complete lattices under operations for left annihilators given by:

$$I \vee J = \mathrm{L}(\mathrm{R}(I+J)) \quad \text{and} \quad \wedge_{k\in K} I_k = \cap_{k\in K} I_k.$$

Analogous formula is for right annihilators.

In several papers QF-algebras and other algebras with $\mathcal{I}_l(A) = \mathcal{A}_l(A)$ were investigated. Algebras A with strange properties of the lattice $\mathcal{A}_l(A)$ were also studied.

In this talk we are going to present some new results about $\mathcal{A}_l(A)$. In particular, for every finite lattice L we indicate a commutative algebra A with $L \subseteq \mathcal{A}_l(A)$ as a sublattice.

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On a generalization of Armendariz rings

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A ring R is called an *Armendariz ring* if whenever the product of two polynomials over R is zero, then the products of their coefficients are all zero, that is, for any $f = \sum_{i=0}^{m} a_i x^i, g = \sum_{j=0}^{n} b_j x^j \in R[x]$,

$$(*) \quad \textit{if } fg = 0, \textit{ then } a_i b_j = 0 \textit{ for all } i, j.$$

The name of these rings honors E. P. Armendariz, who noted in [1] that all reduced rings satisfy this condition. Y. Hirano observed in [2] that condition $(*)$ hides a remarkable connection between the right annihilators of R and those of $R[x]$. Namely, a ring R is Armendariz exactly when the map assigning to any right annihilator A of R the set $A[x]$ of polynomials with coefficients in A is a bijection onto the right annihilators of $R[x]$. Armendariz rings and their numerous generalizations (see [4]) are extensively studied by many authors.

Semi-Armendariz rings were introduced in [3] as a new generalization of Armendariz rings. For a positive integer n, a ring R is said to be an *n-semi-Armendariz ring* provided that if $f = a_0 + a_1x + \cdots + a_mx^m \in R[x]$ satisfies $f^n = 0$, then $a_{i_1}a_{i_2}\cdots a_{i_n} = 0$ for any choice $\{i_1, i_2, \ldots, i_n\} \subseteq \{1, 2, \ldots, m\}$. A ring R is called a *semi-Armendariz ring* if R is n-semi-Armendariz for any positive integer n.

In this talk we will discuss properties of n-semi-Armendariz rings and identify some n-semi-Armendariz subrings of matrix rings. In particular, we will answer a question posed in [3].

This is joint work with Ryszard Mazurek.

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On finite groups with the basis property

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All groups considered here, usually G, are finite. If $X \subseteq G$ is a subset then $\langle X \rangle$ is the subgroup of G generated by X. Let $\Phi(G)$ denotes the Frattini subgroup of G.

There exist several notions of independence in algebra, in particular in group theory (see [2]). Here a subset $X \subseteq G$ is said to be:

- *g-independent* if $\langle Y, \Phi(G) \rangle \neq \langle X, \Phi(G) \rangle$ for every proper subset $Y \subset X$;
- *a g-base of G,* if X is a g-independent generating set of G.

Groups with all g-bases of the same cardinality are known as $\mathcal{B}$*-groups. Groups with the basis property,* that is groups in which every subgroup is a $\mathcal{B}$-group, are studied from several years (see [3], [8] and [5]). These groups were recently described in [1] and [6]. This class is rather narrow: it contains only p-groups and some cyclic q-extensions of them.

If only generators of prime power orders are considered, then an analogue of property $\mathcal{B}$ was denoted in [6] by $\mathcal{B}_{pp}$ and an analogue of the basis property was named the *pp-basis property.* Due to Burnside Basis Theorem p-groups have not only the basis property, but also the pp-basis property.

All cyclic q-extensions of p-groups with the pp-basis property are described in [6] and [7]. Many of them are not with the basis property. The describing of all $\mathcal{B}_{pp}$-groups can be difficult, because we do not know whether there exist a simple nonabelian group with the property $\mathcal{B}_{pp}$ (see [6] and Problem 18.52 in [4]).

During the talk we are going to present some older results and those from [6] and [7] about groups with basis property and with pp-basis property. We will also mention some newer facts about such groups.

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Continuant polynomials

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Continuant polynomials appear as numerators and denominators of the convergents of a continued fraction. P.M. Cohn introduced their noncommutative analogue in order to study the group $GL_2(R)$ of an arbitrary ring R. Some of these results will be mentioned and extended. In the talk relations with right euclidean pairs, quasi-euclidean rings will be presented. Separating the set of continuant polynomials into two families will give interesting graphs emphasizing the leapfrog structure of these polynomials. Relations with Fibonacci sequences and polynomials as well as with tiling will be shown. Generalizations of the continuant polynomials will then appear naturally.

The talk is based on a joint work with A. Facchini.

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On generalizations of Dedekind groups

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A number of authors studied the structure of a finite group G under the assumption that some of its subgroups are well located in G.

Let G be a finite group. Recall that subgroups A and B of G permute if $AB = BA$. A subgroup H is said to be a permutable subgroup of G if H permutes with every subgroup of G. A subgroup H is said to be an s-permutable subgroup of G if H permutes with every Sylow subgroup of G.

T-groups, or groups in which the normality is a transitive relation are a subclass of the class of finite PT-groups, or groups in which permutability is transitive. This class is contained in the class of PST-groups, or groups in which permutability with Sylow subgroups is transitive. These classes have been extensively studied with a lot of characterizations available ([1]). During the talk we are going to present some older results and those from [2], [3] and [4] on groups from these classes. We will also mention some newer facts about such groups.

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Idempotents and clean elements in ring extensions

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The aim of the talk is to present some results on idempotents and clean elements of ring extensions $R \subseteq S$, where S stands for one of the rings:
$R[x_1, x_2, \ldots, x_n]$, $R[x_1^{\pm 1}, x_2^{\pm 1}, \ldots, x_n^{\pm 1}]$, $R[[x_1, x_2, \ldots, x_n]]$.

In particular, criterions for an idempotent of S to be conjugate to an idempotent of R will be presented. Some applications will be given.

It is known that the polynomial ring is never a clean ring. A description of the set of all clean elements in the polynomial $R[x]$ over a 2-primal ring (i.e. $R/B(R)$ is a reduced ring, where $B(R)$ is the prime radical of R) will be presented. It appears that, in general, the description of the set of all clean elements of $R[x]$ is related to the Koethe conjecture.

The talk is based on a joint work with P. Kanwar and A. Leroy.

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Rota-Baxter operators on skew generalized power series rings

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Let A be an associative algebra over a unitary commutative ring K, and let $P : A \to A$ be a K-linear map. The pair (A, P) is called a *Rota-Baxter K-algebra of weight $\lambda \in K$* if

$$P(x)P(y) = P(P(x)y) + P(xP(y)) + \lambda P(xy) \quad \text{for all } x, y \in A.$$

Such a linear operator P is called a *Rota-Baxter operator of weight λ*.

Rota-Baxter algebras have been studied since the 1960s. They appear in many areas of pure and applied mathematics, providing a unified approach to various problems of these different areas. For a short overview of the history and applications of Rota-Baxter algebras we refer to the article [1], and for more detailed information on various aspects of Rota-Baxter algebras to the monograph [2].

Of particular importance are Rota-Baxter operators of weight -1, which will be called simply *Rota-Baxter operators*. A well-known example of a Rota-Baxter operator is the function

$$(*) \qquad P\Big(\sum_{n=m}^{\infty} a_n x^n\Big) = \sum_{n<0} a_n x^n,$$

defined on the algebra $A = K[[x, x^{-1}]]$ of Laurent series. This operator is important because of its role in the renormalization of quantum field theory. Since the function $(*)$ leaves these summands $a_n x^n$ of the Laurent series $\sum_{n=m}^{\infty} a_n x^n$ for which $n < 0$ and kills all other summands $a_n x^n$, the function P is called the *cut-off operator* (at the point 0).

For any commutative ring R, the Laurent series ring $R[[x, x^{-1}]]$ is a special case of so-called generalized power series rings $R[[S]]$, where S is a strictly ordered commutative monoid. Since an analogue of the cut-off operator can still be defined on the ring $R[[S]]$, in [3] Guo and Liu considered (and answered) the natural question of when the cut-off operator is a Rota-Baxter operator on $R[[S]]$.

An analogous question can be considered for a more general ring construction, called the skew generalized power series ring $R[[S, \omega]]$, where R is a ring, S is a strictly ordered monoid and $\omega\colon S \to \mathrm{End}(R)$ is a monoid homomorphism. The skew generalized power series ring construction (introduced in [5]) is a common generalization of (skew) polynomial rings, (skew) Laurent polynomial rings, (skew) power series rings, (skew) Laurent series rings, (skew) monoid rings, (skew) Mal'cev-Neumann series rings, and generalized power series rings. In this talk we will characterize the subsets T of S for which the cut-off operator with respect to T is a Rota-Baxter operator on the ring $R[[S, \omega]]$.

This talk is based on the paper [4]. The obtained results provide a large class of noncommutative Rota-Baxter algebras, extending the results of [3] to the noncommutative case.

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Conjugacy classes of left ideals of an associative algebra

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Let A be a finite dimensional unital algebra over a field K and let $C(A)$ denote the set of conjugacy classes of left ideals in A. The set $C(A)$ can be considered as a semigroup under the natural operation induced from the multiplication in A. The finiteness of this semigroup was proved to be strongly related to the finite representation type property of finite dimensional algebras. Moreover, some properties of the algebra A can be recognized by the semigroup $C(A)$. In particular, one can ask if the isomorphism of semigrups $C(A)$ and $C(B)$ for finite dimensional K-algebras A, B implies, that these algebras are isomorphic, provided that the field K is algebraically closed? We give some recent results concerning this problem. This is joint work with my PhD advisor Prof. Jan Okninski.

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Remarks on algebraic and geometric properties of the spark of a matrix

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Let $C_1, \dots, C_n \in \mathbb{F}^m$ be the columns of an $m \times n$ matrix A over a field $\mathbb{F}$. The spark of A is defined to be the infimum of the set of all positive integers ℓ such that

$$\exists\, j_1, \dots, j_\ell \in \{1, \dots, n\} : \begin{cases} j_1 < \dots < j_\ell, \\ C_{j_1}, \dots, C_{j_\ell} \text{ are linearly dependent.} \end{cases}$$

The spark of a matrix plays a quite important role in the mathematical theory of Compressed Sensing. In the talk, we will recall some known algebraic and geometric results concerning the spark, and provide a few new examples and observations.

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On finite matroid groups

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All groups considered here are finite. Let $\Phi(G)$ denote the Frattini subgroup of a group G, that is the set of nongenerators of G. A subset X of a group G is called *g-independent*, if there is no proper subset $Y \subset X$ such that $\langle X, \Phi(G)\rangle = \langle Y, \Phi(G)\rangle$. We say that the group G has *property* $\mathcal{B}$ (is a $\mathcal{B}$-*group* for short), if all its minimal generating sets have the same cardinality. If additionally every g-independent subset of G can be extended to a minimal generating set of G, then G is called a *matroid group.* In view of Burnside Basis Theorem, it is obvious that p-groups are matroid. In [2] the full characterization of matroid groups was provided. A small variation of this characterization was done in [1].

The talk will focus on the description of matroid groups and its properties. We also mention groups satisfying analogous conditions, but only for sets of prime power order generators.

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On Milnor laws

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One of the problems in group theory is the classification of group laws and varieties of groups. This classification is far from being complete. Important classes of group laws are $\mathfrak{R}$-laws and Milnor laws. A binary law $w \equiv 1$ is called an $\mathfrak{R}$-law (a restrained law) if every finitely generated group satisfying $w \equiv 1$ has a finitely generated commutator subgroup ([2]). For example Engel laws and positive laws are $\mathfrak{R}$-laws. A binary law is called a Milnor law if the variety of groups satisfying this law does not contain any of varieties $\mathfrak{A}_p\mathfrak{A}$ where p is a prime number. Here $\mathfrak{A}_p\mathfrak{A}$ is the variety of groups G that have a normal abelian subgroup N such that G/N is abelian. There are several equivalent conditions for Milnor laws ([3], [4], [5], [1]). But they are not constructive. In my talk I show several definitions of $\mathfrak{R}$-laws and Milnor laws and connections between them. I discuss constructive conditions for a binary word to be a Milnor law and conditions for Milnor laws to have additional properties. I discuss also open problems in this area.

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Session: Computer Science

The standard rough inclusion function as a basis for similarity indices

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Rough inclusion functions (RIFs for short) are mappings used in rough set theory [4–7] to measure the degree of inclusion of a set of objects (*information granule* in particular) in a set of objects (information granule). RIFs are supposed to comply with axioms of rough inclusion, formulated as axioms of L. Polkowski's and A. Skowron's *rough mereology* [9–12]. More precisely, a RIF upon a non-empty set U (the universe of objects) is a mapping $\kappa : \wp U \times \wp U \mapsto [0, 1]$, assigning to any pair of sets (X, Y) of elements of U, a number $\kappa(X, Y)$ from the unit interval $[0, 1]$ interpreted as the degree to which X is included in Y, and such that the conditions $\mathrm{rif}_1(\kappa)$ and $\mathrm{rif}^*_2(\kappa)$ are satisfied, where

$$\mathrm{rif}_1(\kappa) \overset{\mathrm{def}}{\Leftrightarrow} \forall X, Y \subseteq U.(\kappa(X, Y) = 1 \;\Leftrightarrow\; X \subseteq Y),$$
$$\mathrm{rif}^*_2(\kappa) \overset{\mathrm{def}}{\Leftrightarrow} \forall X, Y, Z \subseteq U.(\kappa(Y, Z) = 1 \;\Rightarrow\; \kappa(X, Y) \le \kappa(X, Z)).$$

As a tool, RIFs are useful not only in rough set theory but also in a more general framework of granular computing [8, 13]. Among RIFs the *standard* RIF is of a special importance for application.

The notion of the standard RIF, $\kappa^{£}$, is closely related to conditional probability. In logic, J. Łukasiewicz employed the same idea to calculate the probability of truth of implicative formulas [3]. For a finite U, $\kappa^{£}$ is defined by

$$\kappa^{£}(X, Y) \overset{\mathrm{def}}{=} \begin{cases} \frac{\#(X \cap Y)}{\# X} & \text{if } X \neq \emptyset, \\ 1 & \text{otherwise}, \end{cases} \tag{1}$$

where X, Y are any subsets of U and $\# X$ denotes the number of elements of X.

The *similarity indices* considered here serve in *cluster analysis* as a tool to measure the degree of similarity between two clusterings (and indirectly between the clustering methods producing these clusterings). In our work we consider 22 different similarity indices known from the literature on classification and cluster analysis, studied by A. N. Albatineh, M. Niewiadomska-Bugaj, and D. Michalko in [1] in detail.

Given a set U_0 of $m > 0$ data points to be grouped by clustering methods A_1 and A_2. Let C_1 and C_2 be clusterings, i.e. partitions of U_0 generated by A_1 and A_2, respectively. Typically, the degree of similarity between C_1 and C_2 (and indirectly between A_1 and A_2) is calculated this way or another, taking into account the number of pairs of data points that are put into the same cluster and/or the number of pairs of data points that are put into different clusters by A_1 and A_2.

As it turned out, all the similarity indices studied in [1] can be reformulated in terms of the standard RIF $\kappa^{\pounds}$ or two other RIFs, κ_1 and κ_2. In this presentation we first report on these results [2]. Next, we show that $\kappa^{\pounds}$ alone is sufficient for generation of all 22 similarity indices. Finally, we present several new similarity indices based on the standard RIF.

Keywords: standard rough inclusion function, similarity index, cluster analysis, rough set theory, granular computing.

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On the SAT-based verification of communicative commitments

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We present CDCTL*K, a temporal logic to specify knowledge, correct functioning behaviour, and different social commitments in multi-agent systems (MAS). We interpret the formulae of the logic over models generated by Communication Deontic Interpreted Systems (CDIS). Furthermore, we investigate a SAT-based bounded model checking (BMC) technique for the existential fragments of CDCTL*K (ECDCTL*K) and for CDIS. Finally, we exemplify the use of the technique by means of the NetBill protocol, a popular example in the MAS literature related to modelling of business processes.

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Similarity-based searching in structured spaces of music information *

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The study is focused on structural analysis of musical pieces, based on specialized grammars covering music information. The analysis may be performed on a single musical piece, where one searches for leitmotifs. A comparative analysis may also be performed, where a comparison of rhythmic and melodic motives is performed. Rhythmic motives are regarded here as patterns of notes' durations of various length, starting at length one, that allows for a simple distribution of notes length for a piece. The operators allow to relativize rhythmic and melodic motives of length 2 and more so they became more universal. In this paper we focus on searching for transformed and non-transformed motives, analysing melodic and rhythmic sequences, structure discovery and comparative analysis of musical pieces.

Keywords: music information, searching, knowledge discovery, knowledge understanding, syntactic structuring, querying

Optimization problems and methods applicable in intelligent tourist travel planners

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The paper describes the project of innovative software component – LOGTRAVEL which can be applied as a logistic toolbox for e-tourism systems called Tourist Travel Planners (TTP). Functionalities of LOGTRAVEL supports planning and organization of cheap and attractive touristic travels which tourists can use in TTP system. The component includes solutions of many variants and extensions of Orienteering Problem which enable a generation of an optimal trip satisfying variety traveler preferences. The paper has the survey character and is to the definition of the problems and their application and does not present solutions for them.

Keywords: Tourist Travel Planners, Orienteering Problem, Point of Interest

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Combining genetic algorithm and path relinking for solving Orienteering Problem with Time Windows

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Path relinking is a search strategy that explores trajectories connecting two solution to generate new best solutions. In this article two options combining path relinking and genetic algorithm are investigated: one introduced a path relinking between selected generations of population, and the other applies path relinking when genetic algorithm solution trapped in a local optimum. These two strategies are applied with the genetic algorithm solving orienteering problem with time windows. Experiments carried out on benchmark instances show that proposed methods obtain better solutions than standard genetic algorithm.

Keywords: orienteering problem with time windows, genetic algorithm, local optimum, path relinking

An approach to making decisions with metasets

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The metaset is a new approach to sets with partial membership relation. Metasets are designed to represent and process vague, imprecise data, similarly to fuzzy sets or rough sets. They make it possible to express fractional certainty of membership, equality, and other relations. In this paper we demonstrate an example of the application of first-order metasets to solving the problem of finding the most appropriate holiday destination for a tourist, taking his preferences into account. The imprecise idea of 'a perfect holiday destination' is represented as a metaset of places whose membership degrees in the metaset are interpreted as their qualities. Client preferences are functions which enable real-number evaluation of the subjective rating of a given destination.

Keywords: metaset, partial membership, set theory

Action rules mining in laryngological disorders

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Action rule is an implication rule that shows the expected change in a decision value of an object as a result of changes made to some of its conditional values. An example of an action rule

is "patients are expected to control their health regularly if they receive an information about free medical tests once a year". In this case, the decision value is the health status, and the condition value is whether the information is sent to the patient. The type of action that can be taken by the medical centre is to send out information about free medical tests. The action rule discovery algorithms mainly build action rules from existing classification rules. This paper discusses a method that generates the shortest action rules directly from a decision system. In particular, the algorithm can be used to discover rules from an incomplete decision system where attribute values are partially incomplete. As one of the testing domains for our research we take new system which is forming through a collaboration between the Department of Mechanics and Computer Science at Bialystok University of Technology and physicians at the Department of Clinical Phonoaudiology and Logopedics at Medical University of Bialystok and Laryngologic Medical Centre. This system was designed for gathering and processing clinical data on patients with throat disorders, and mining action rules will be help to construct the decision-support module.

Do there exist complete sets for promise classes?

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Complexity theorists distinguish complexity classes that are defined syntactically and classes that are defined semantically (promise classes). The rule of thumb is that syntactic classes possess complete sets, while promise classes do not.

In the talk we will present the notion of a complexity class representable in a proof system. We will show that a promise class C has complete sets if and only if there exists a proof system for some language L in which C is representable.

Additionally, we will discuss the connection between the existence of optimal proof systems and the existence of complete sets for promise classes.

Keywords: computational complexity, optimal proof systems, promise classes

Dialogue as a game

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Dialogue games have found recent application in philosophy, computational linguistics and Artificial Intelligence. In philosophy, they have been used to study fallacious reasoning and to develop a game-theoretic semantics for various logics. In linguistics, they have been used to explain sequences of human utterances, with subsequent application to machine-based natural language processing and generation. Within Computer Science and Artificial Intelligence, they have been applied to modelling complex human reasoning and to requirements specification for complex software systems. Our research focus on the application of formal dialogue games for agent communication and interaction using argumentation.

In the presentation we show how to construct a dialogue game and we present one of the formal dialogue systems for natural languages called Lorenzen-Prakken Natural Dialogue, LPND. This system was designed for attacking inferences in formal natural dialogues. To this purpose both natural and formal dialogues were described using one language. In [4], we have reconstructed Lorenzen's dialogical logic [2] and we have proposed coherent dialogue protocol using the tradition of persuasion dialogue games as specified by Prakken [3]. The resulting system is called Lorenzen Natural Dialogue (LND). In [1] we introduced Prakken Natural Dialogue (PND) in which players are allowed to commit formal fallacies, i.e. fallacies that use schemes which are not equivalent to valid formulas of the underlying logic. PND allows for modelling dialogues in which inference rules used by players are publicly declared and can be challenged. Now, we join these two approaches.

Finally we demonstrate a software tool which implements the protocol LND and present the rules for embedding LND system into PND system. The implementation was written in Java language, which will facilitate further development of the application, as well as software portability. The participants in the dialogue and the game manager are implemented as separate classes distributed over the network. The aim of this implementation is to simulate a quasi-natural dialogue in order to make its analysis as well as to support making decision during such a game.

Keywords: protocol for dialogue games, natural dialogue, formal fallacy

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Multi-valued De Morgan gate

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Bayesian networks offer a useful framework for reasoning in uncertain problem domains. Maaskant and Druzdzel in [1] introduced the De Morgan gate, an Independence of Causal Interactions model, that is capable of modeling opposing influences, composed of noisy OR and noisy AND gates and their negative counterparts. The main goal of this talk is to define the De Morgan gate for multi-valued variables, which combined influences noisy MAX and noisy MIN gates and their negative counterparts. To achieve this goal I introduced negative influences models for MAX and MIN gates and described how all four gates simultaneously influence the effect. I implemented defined model to test its usefulness for elicitation of conditional probability distributions in practice. I present the way of obtaining numerical parameters for multi-valued De Morgan model, the results of the experiments and conclusions.

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Comparison of different graph weights representations used to solve the Time-Dependent Orienteering Problem

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The paper compares three types of network weights representations applied to solve the Time-Dependent Orienteering Problem (TDOP). First variant uses real, time-dependent weights, second variant uses mean weights and the third is a hybrid of the previous two variants. A randomized, local search algorithm was used in this comparison and tests were conducted on real public transport network of Białystok. The results show importance of both detail and general properties of network topology when solving TDOP.

Keywords: time-dependent orienteering problem, public transport network, mean weights, time-dependent weights, hybrid weights, comparison

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Session: Computer-Assisted Formalization of Mathematics – In Memoriam of Andrzej Trybulec

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Formal proofs of hypergeometric sums (dedicated to the memory of Andrzej Trybulec)

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Algorithmic methods can successfully automate the proof, and even the discovery, of a large class of identities involving sums of hypergeometric terms. In particular, the Wilf-Zeilberger (WZ) algorithm is a uniform framework for a substantial class of hypergeometric summation problems. This algorithm can produce a rational function *certificate* that can, on the face of it, be used to verify the result by routine algebraic manipulations, independently of the working of the algorithm that discovered it. It is therefore very natural to consider using this certificate to produce, by automated means, a rigorous deductive proof in an interactive theorem prover. However, naive presentations of the WZ method tend to gloss over trivial-looking but rather knotty questions about zero denominators, which makes their rigorous formalization tricky and their ultimate logical justification somewhat obscure. We describe how we have handled these difficulties to produce rigorous WZ proofs inside the HOL Light theorem prover.

Budget imbalance criteria for auctions: a formalized theorem

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We present an original theorem in auction theory: it specifies general conditions under which the sum of the payments of all bidders is necessarily not identically zero, and more generally not constant. Moreover, it explicitly supplies a construction for a finite minimal set of possible bids on which such a sum is not constant. In particular, this theorem applies to the important case of a second-price Vickrey auction, where it reduces to a basic result of which a novel proof is given. To enhance the confidence in this new theorem, it has been formalized in Isabelle/HOL: the main results and definitions of the formal proof are reproduced here in common mathematical language, and are accompanied by an informal discussion about the underlying ideas.

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Formalization of Fundamental Theorem of Finite Abelian Groups in Mizar

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In this paper, we report the formalization of fundamental theorem of finite abelian groups, which describes the structure of finite abelian group. As the theorem is a basic tool to study on finite abelian groups, it is expected to be widely applied to formalizations in a variety of fields such as number theory or cryptology in the future Mizar articles.

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Feasible analysis of algorithms with Mizar

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In this paper we present an approach to the analysis of algorithms with Mizar. It consists in the Mizar formalization of abstract concepts like algebra of instructions, execution function, termination and their substantiation in a model with integers as the only data type and in models with abstract data types. The proof of correctness of the algorithm Exponentiation by Squaring is described.

The approach is aimed at creating Mizar tools for verification of real programs, but primarily to be used in teaching.

Formal characterization of almost distributive lattices

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Almost distributive lattices (ADL) are structures defined by Swamy and Rao in 1981 as the generalization of ordinary distributive lattices by removing certain conditions from well-known axiomatization of these. As distributive lattices are pretty well present in the Mizar Mathematical Library (MML), we decided to give also formal characterization of ADL in terms of Mizar attributes and clusters. Many of the lemmas and counterexamples can be obtained semiautomatically with the help of external provers (e.g. Prover9 and MACE), also internal Mizar utilities can improve human work in this area. We formalized crucial parts of the chosen papers of Rao (some of the are pretty recent), obtaining also the characterization of the class of generalized almost distributive lattices.

Formalizing cryptographic algorithms in Mizar with the Enigma example

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Cryptography is one of the most pressing problems of modern computer science. The amount of private and confidential information both being stored digitally and sent through the Internet is immense, therefore the security is of utmost importance. The formalization of the most commonly used cryptographic algorithms is meant to increase our understanding of the possible threats to encryption systems [6, 11, 10].

The Mizar Mathematical Library, the repository of formal texts written and verified using the Mizar proof assistant [8, 13, 1], already includes articles on the subject of cryptography and cryptology [4, 5, 9, 2].

My focus is on building a library of formalized cryptographic algorithms in the Mizar language, both to enrich the MML and to further the studies on the subject. The presentation will show my work on the formalization of the Enigma, a cipher machine used by Nazi Germany during World War II, and was a combination of mechanical and electrical subsystems. It utilized a set of rotors, that were used for encryption. The code was broken by the Polish Cipher Bureau, which consisted mostly of mathematicians, including Marian Rejewski, Jerzy Różycki, and Henryk Zygalski [7, 12, 3].

The Enigma is interesting both from the cryptographic and the historical point of view.

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Towards standard environments for formalizing mathematics

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Though more and more advanced theorems have been formalized in proof systems their presentation still lacks the elegance of mathematical writing. The reason is that proof systems have to state much more details – a large number of which is usually omitted by mathematicians. In this paper we argue that proof languages should be improved into this direction to make proof systems more attractive and usable – the ultimate goal of course being a like-on-paper presentation. We show that using advanced Mizar typing techniques we already have the ability of formalizing pretty close to mathematical paper style. Consequently users of proof systems should be supplied with environments providing and automating these techniques, so that they can easily benefit from these.

Improving legibility of proof scripts based on quantity of introduced labels*

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Formal proof checking systems such as Mizar or Isabelle/Isar can verify the correctness of proof scripts, both easily readable and obscure. However for humans, e.g., those who analyse the main

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idea of a formal proof or redevelop fragments of reasoning to make them stronger, the legibility has substantial significance. Furthermore, proof writers create still more and more complex deductions that cannot be shortened to several steps by any tools currently available. Therefore, it is important to better understand how we can facilitate the work of script readers modifying the order of independent deduction steps or reorganise the proof structure by extracting lemmas that are obtained automatically.

In this paper we present experimental result obtained with a method that improves proof legibility based on human short-term memory and we explore its impact for realisation of other, also popular methods.

Keywords: operations on languages, legibility of proofs, proof assistants

The TPTP typed first-order form with arithmetic: the language and some applications

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The TPTP World is a well established infrastructure supporting research, development, and deployment of Automated Theorem Proving systems. In 2010 the TPTP World was extended to include a typed first-order logic, which in turn enabled the integration of arithmetic. This talk describes the Typed First-order with Arithmetic (TFA) part of the TPTP World – the language syntax and semantics, the arithmetic capabilities, and some applications that use these features.

A heuristic prover for real inequalities

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Comparing measurements is fundamental to the sciences, and so it is not surprising that ordering, bounding, and optimizing real-valued expressions is central to mathematics. A host of computational methods have been developed to support such reasoning, using symbolic or numeric methods, or both. Interactive theorem provers like Isabelle and HOL Light now incorporate various such methods, either constructing correctness proofs along the way, or reconstructing them from appropriate certificates. Such systems provide powerful tools to support interactive theorem proving. But, frustratingly, they often fail when it comes to fairly routine calculations, leaving users to carry out explicit calculations painstakingly by hand. Consider, for example, the following valid implication:

$$0 < x < y,\ u < v \ \Rightarrow\ 2u + exp(1 + x + x^4) < 2v + exp(1 + y + y^4)$$

The inference is not contained in linear arithmetic or even the theory of real-closed fields. The inference is tight, so symbolic or numeric approximations to the exponential function are of no use. Backchaining using monotonicity properties of addition, multiplication, and exponentiation might suggest reducing the goal to subgoals $2u < 2v$ and $exp(1 + x + x^4) < exp(1 + y + y^4)$, but this introduces some unsettling nondeterminism. After all, one could just as well reduce the goal to

$2u < exp(1 + y + y^4)$ and $exp(1 + x + x^4) < 2v$, or one of various other options. In fact, few current methods prove this implication.

And yet, the inference is entirely straightforward. With the hypothesis $u < v$ in mind, notice that the terms $2u$ and $2v$ can be compared; similarly, the comparison between x and y leads to comparisons between x^4 and y^4, then $1 + x + x^4$ and $1 + y + y^4$, and so on.

We propose a method for solving such real-valued inequalities based on this style of heuristically guided forward reasoning, using properties of addition, multiplication, and the function symbols involved. As is common for resolution theorem proving, we try to establish the theorem above by negating the conclusion and deriving a contradiction. We then proceed as follows:

- Put all terms involved into a canonical normal form. This enables us to recognize terms that are the same up to a scalar multiple, and up to associativity and commutativity of addition and multiplication.
- Iteratively call specialized modules to learn new comparisons between subterms, and add these new comparisons to a common "blackboard" structure, which can be accessed by all modules.

The theorem is verified when any given module derives a contradiction using this common information. The procedure fails, on the other hand, when none of the modules can learn anything new.

We see that the method is far from complete, and may not even terminate. On the other hand, it is flexible and extensible, and easily verifies many heterogeneous inferences that are not obtained using more principled methods. As a result, it provides a useful complement to more conventional approaches. A prototype implementation in Python indicates the utility and efficiency of our method. In the future it will be integrated as a proof-producing tactic in interactive theorem provers.

Natural deduction in the Metamath proof language

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The Metamath proof language is a system for writing formal proofs in a Hilbert system. In this talk I present an overview of the language and draw some comparisons to the Mizar proof language. I also challenge the common preconception that Gentzen systems (natural deduction proofs) like Mizar are in some sense inherently more efficient than their Hilbert counterparts, with examples drawn from the many successful proof projects undertaken in the Metamath database. Certainly it is the case that most mathematicians think most naturally within a natural deduction framework, but I demonstrate that one can emulate any natural deduction proof effectively in a Hilbert system.

The Deduction Theorem is often used as the justification for the use of a natural deduction system. I will present a procedure for producing deduction proofs that is more efficient than many standard proofs of the Deduction Theorem, using only a constant number of extra steps and stable under multiple application.

On logical and semantic investigations in Kyiv school of automated reasoning

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The talk is devoted to the brief description of the logical and semantic approaches being developed in the Kyiv school of automated reasoning. This approach can be traced back to 1970, when academician V. Glushkov initiated research on automated theorem proving in mathematics, which is known as the Evidence Algorithm programme, EA, having many common theses with the Polish Mizar project.

Historically, the time span of automating theorem proving in the EA-style can be divided into four stages: 1962–1969, when the first steps were made in this area; 1970–1982, when the studies were carried out in accordance with EA, which led to the implementation of the Russian-language System for Automated Deduction, SAD; 1983–1992, when the resolution technique used in the Russian SAD was improved, and 1998 – present, when, after a certain period of the absence of any research in the EA-style, Glushkov's position has been revised in the light of new paradigms and advances in automated reasoning, which led to the appearance of the English-language SAD system[1].

Naturally, in all these stages, a significant attention has being paid to the study and development of logic and semantics as well as a technique for logical inference search.

Carried out first at the Institute of Cybernetics of NASU, these investigations then in 1987 actually moved to the Faculty of Cybernetics of the National University of Kyiv, where now they are developing in two directions. The first direction is the traditional one, centered mainly on the construction of proof methods in various first-order logics. The second direction aims to develop logics oriented on semantic models of programs.

The following results have been obtained within the first direction: (1) new calculi and methods including resolution- and paramodulation-type techniques as well as modifications of Maslov's Inverse Method have been constructed; (2) research on inference search in various first-order classical and non-classical logics oriented to their implementation in automated reasoning systems, in particular, in the English SAD has been made; (3) on this basis, the logical "engine" of SAD has been modified.

The following results have been obtained within the second direction: (1) a hierarchy of algebra-based program model has been constructed; (2) new program-oriented logics of partial predicates and functions based on principles of compositionality and nominativity have been developed; (3) sound and complete sequent calculi have been constructed for these logics; (4) algorithms for reduction of the satisfiability problem in these logics to the satisfiability problem in classical logic have been developed.

It is expected that the obtained results will give a possibility to extend SAD with more expressive logical languages and more powerful reasoning tools.

[1] Details can be found in: Lyaletski A., Morokhovets M., Paskevich A. Kyiv school of automated theorem proving: a historical chronicle, In: Logic in Central and Eastern Europe: History, Science, and Discourse. – University Press of America, 2012. – P. 431-469.

On an alternative approach to Skolemising

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As it is well-known the problem of finding appropriate terms to substitute for the individual variables (when the rules for classical quantifiers are applied) is very important during automated theorem proving. One way to overcome this problem is to introduce unification. Another problem connected with the unification of terms is the problem of removing classical quantifiers from formulas. Skolemising is one of the well known methods that allows to remove existential quantifiers and to introduce constants or proper functions.

It appears that sometimes an alternative approach to Skolemising [1] can be used as very suitable, for example, when we deal with automated theorem proving in a given logic of programs ([4]) (the task is not so easy for the formula of the type $\forall_x \exists_y ((y := \varphi(y)) M\alpha(x,y))$ where M is a program). The main idea of the mechanism is as follows:

- first, the tree ordering of the formula, denoted by $<$, is introduced; the relation describes dependence between terms and variables which have been introduced during process of building the diagram of the formula,
- next, during the process of unification for each substitution σ(which substitutes terms for variables) a certain relation $< \bullet$ is introduced with some restriction: the transitive closure of the union of the relation $<$ and the relation $< \bullet$ does not lead to a cycle.

The steps of proving with unification (in Gentzen -type) is presented below:

Step 1. Turn the input formula into prefix form.

Step 2. While applying the rules for classical quantifiers to build unification diagram for a given decomposition tree of the formula. The unification diagram is the ordered pair $< \mathcal{D} \cup \mathcal{Z}, \succ >$, where

- $\mathcal{D}$ is a set of variables (called d-variables), which are some kind of imitations (on this level of proving we do not know which terms will be proper) of terms that are introduced by rules for existential quantifiers,
- $\mathcal{Z}$ is a set of variables (called z-variables), which are introduced by rules for general quantifiers,
- $\succ$ is a relation such that for arbitrary variables $t_1, t_2 \in \mathcal{D} \cup \mathcal{Z}$ it holds that $t_1 \succ t_2$ if and only if the quantifier $\mathbf{Q}_2$ introducing the variable t_2 is in the immediate range of the quantifier $\mathbf{Q}_1$ introducing the variable t_1.

Step 3. While checking whether the given sequent Π is fundamental or not if only the sequent is not fundamental and the set $\mathcal{D} \cup \mathcal{Z} \neq \emptyset$, do the unification in the following way:

look for pair of formulas $\alpha, \beta \in \Pi$ such that
$$\alpha = \varrho(\tau_1, \ldots, \tau_n) \quad \text{and} \quad \beta = \neg\varrho(\tau_1', \ldots, \tau_n'),$$
where $\tau_1, \ldots, \tau_n, \tau_1', \ldots, \tau_n'$ are terms and ϱ is n-ary predicate and find such substitution σ that

- $\sigma(\tau_1) = \sigma(\tau_1'), \quad \ldots \quad , \quad \sigma(\tau_n) = \sigma(\tau_n')$ and

– after adding to the unification diagram for the substitution σ two relations: equivalence one $\sim$ and additional one $\ll$ that are defined in the following way:
  – if $\sigma = [d_1 \setminus d_2]$, then $d_1 \sim d_2$,
  – if $\sigma = [d_1 \setminus \tau]$ and τ is not d-variable then $d_2 \ll d_1$ and $z \ll d_1$ for each d-variable d_2 occurring in τ and for each variable z occurring in τ;
  – $\sim$ satisfies the conditions of equivalence relation and if $d_2 \sim d_1$ and $z \ll d_1$ then also $z \ll d_2$,

the transitive closure of the union of the relation $\succ$ and the relation $\ll$ does not lead to a cycle.

The alternative approach to Skolemising will be presented in the context of Davis's obviousness criterion [2]. The criterion identifies obvious inferences with those correct infereces for which Herbrand proof can be given involving no more than one substitution instance of each clause. A similar criterion for obvious inferences was adopted in the MIZAR MSE system by Trybulec [3]. Some examples will be discussed.

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Implementing a parser for the WS-Mizar language

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The Mizar language [2] is devised to encode mathematical formulas and their proofs in a form that is as much as possible readable for humans, which makes its syntax rather complex. On the other hand, it is equally important that the texts written in the Mizar language should be effectively processed by the set of programmes included in the Mizar system distribution. For these reasons, it is a non trivial task to implement new independent utilities for parsing Mizar texts. To tackle this problem, a new language has been proposed – WS-Mizar (Weakly Strict Mizar, WSM) [1], which is less complex than the original Mizar language from a programmer's point of view, while there is a software tool available that can translate any text written in Mizar into its WS-Mizar representation. The goal of this talk is to present how the grammar of WS-Mizar was formally created based on the available grammar of original Mizar, which enabled the implementation of a dedicated parser independent from the one built into the Mizar system.

The Mizar language is described by its grammar available in the system's distribution (files `syntax.txt` and `syntax.xml`) as well as on the project's website[1]. This grammar, however, contains some ambiguities stemming from making the language as close as possible to the informal way of writing mathematical proofs, which must be solved by a working implementation. These ambiguities

[1] `http://mizar.org/language/mizar-grammar.xml`

make it practically difficult to implement an external utility for analyzing Mizar texts based only on this grammar, and so the WS-Mizar language was proposed. The name „Weakly Strict Mizar" should indicate that the language is not completely strict in the sense of containing full semantic representation, but it is strict enough to enable parsing the text without any particular knowledge of how the text is internally processed by the Mizar system. It is also worthwile to note another language mentioned in [1], namely MS-Mizar (More Strict Mizar, MSM), which is a next step in representing more semantic information contained in Mizar texts. Implementing a simple parser, however, does not require using MSM.

With the current Mizar system one can generate a WS-Mizar document from a Mizar article using the `wsmparser` tool. When the article is processed, the system creates also a WSX version of the file, which contains its WS-Mizar content represented in an XML-based format. Formal specification of the WS-Mizar grammar was possible thanks to combining the original Mizar grammar with Mizar article's representation stored in MIZ, WSX and WSM files.

A parser for WS-Mizar based on that grammar was implemented using the popular open-source GNU parser generator suit: `flex` and `bison` [3]. The parser's source code is written in the C programming language and compiled with the GCC compiler. The underlying lexer generated by `flex` is augmented by the information extracted from the environment of a Mizar article which needed a separate ad-hoc parsing. The parser is to be used mainly in the GNU/Linux system, but it can also be used within MinGW or Cygwin environments under MS Windows.

The main result of the work described here is the implementation of an independent parser of the WS-Mizar language. This is the first step needed to start developing various external utilities for processing mathematical data contained in Mizar articles. At the same time, the work allowed to sort out some issues in the grammar of the standard Mizar language, which in turn helped to improve the system's documentation.

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Parallelizing Mizar

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This paper surveys and describes the implementation of parallelization of the Mizar proof checking and of related Mizar utilities. The implementation makes use of Mizar's compiler-like division into several relatively independent passes, with typically quite different processing speeds. The information produced in earlier (typically much faster) passes can be used to parallelize the later (typically much slower) passes. The parallelization now works by splitting the formalization into a suitable number of pieces that are processed in parallel, assembling from them together the required results. The implementation is evaluated on examples from the Mizar library, and future extensions are discussed.

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Combining Mizar with Logic2CNF SAT solver

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The talk is devoted to presenting an experimental extension of the MIZAR system employing an external SAT solver to strengthen the notion of obviousness of the MIZAR proof checker. Although its user input language is being developed to resemble standard mathematics as much as possible, the amount of details a user must currently provide to make the system check the text's full logical correctness is still too big. The strength of the proof checking system is the most important factor responsible for maintaining the de Bruijn factor on a low level. Therefore numerous new techniques are being currently developed to make the MIZAR checker stronger. There is also active research on combining MIZAR with external automated theorem provers. Still, the most widely-used and simplest method is based on MIZAR "requirements", which provide a way to implement specific procedures that make the checker handle certain simple mathematical objects, frequently used in typical MIZAR texts. This includes special treatment of Boolean operations on sets, complex arithmetic and the like. Since the MIZAR library is built on top of set theory axioms, the usage of various set-based constructs is ubiquitous in the library. Apart from "articles" devoted to sets *per se*, there are many more abstract ones that heavily use sets for constructing some models or examples (e.g. in geometry, lattice theory or graph theory). Therefore the automation of processing sets is beneficial for most of the current library (enabling to reduce its size) but most importantly for future developments. Hard-coding specific checking of Boolean operations, known as "requirements BOOLE", was implemented quite early in the history of MIZAR development and it remained partial and not very efficient. In this talk we present the extension which exploits the natural correspondence between propositional formulae and Boolean operations on sets in order to eliminate the need of referencing definitions of these operations and all sorts of lemmas based on them in MIZAR proofs.

For ease of implementing the interface needed to interact with the MIZAR proof checker in a way very similar to the interface previously implemented for Gröbner bases computation, we decided to use a MiniSAT[1] variant developed by Edd Barrett, called Logic2CNF[2]. Logic2CNF uses a small input language for logic input from file/stdin. It then converts it to CNF, solves it using built-in MiniSAT and reports the results as assignments of input literal names.

The simple interface we implemented uses the Logic2CNF input language to construct a formula in which each propositional variable represents equality classes made up of all available terms. If a term represents a Boolean operation, the input stream is appended by a corresponding logical formula (e.g. set intersection yields a conjunction in the input and so on). Then, every instance of a negated equality in a given inference (MIZAR's checker is a disprover, so this means testing if some two sets are equal) is checked whether it logically entails the conjunction of all previously stored formulae. This is naturally obtained by negating the entailment and computing its satisfiability with a call to Logic2CNF. From the user's point of view, the proposed extension works automatically and there's no need to directly call the external tool. The Mizar verifier (as well as other MIZAR

[1] MiniSAT, developed by Niklas Eén and Niklas Sörensson, is a minimalistic SAT solver that supports a standard DIMACS input notation. The system is successfully used in a number of other projects, because it is relatively easy to modify, well-documented, highly efficient and designed for integration as a backend to other tools.

[2] Logic2CNF is available for download at http://projects.cs.kent.ac.uk/projects/logic2cnf/trac/wiki/WikiStart.

tools that use its checker) just spawns a new Logc2CNF process whenever it is needed to justify any goal that involves Boolean operations.

Using the standard Mizar tools such as `relprem` (for eliminating unnecessary references) and `trivdemo` (reducing simple proofs to straightforward justifications), `checklab` (detecting unused labels) and `inacc` (removing unused text fragments), the encyclopedic article `XBOOLE_1` containing many facts on Boolean operations of sets can be significantly reduced. In particular, 32 out of 117 theorems become obvious for the checker and simply by removing the unnecessary proofs the article's size can be reduced by 35%. As for the whole library, `relprem` reported in total over 1600 unnecessary references in 22% of all the library articles.

The prototype of the extended Mizar system equipped with the interface to Logic2CNF and pre-compiled for main supported software platforms can be downloaded from the author's web-site: `http://mizar.uwb.edu.pl/~softadm/boolesat`.

Equalities in Mizar

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The usage of extensionality of sets, possibly satisfying some additional properties, by proof checkers will be presented. In particular, we will show how extensionality influences proof tactics and equational calculus. In the paper, we collect short descriptions of Mizar constructions, which have an impact on two basic Mizar modules: Reasoner for supporting definitional expansions, and Equalizer for computing the congruence closure of a given set of equalities.

Session: Control Theory and Dynamical Systems

Getting rid of chaotic population dynamics

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Apparently simple population models given by one-dimensional difference equations can have a complex behaviour, in which population size fluctuates chaotically, as a consequence of intraspecific competition. This complex behaviour appears also in higher-dimensional systems modelling, for example, age-structured populations, and it has been observed experimentally.

We will present some strategies proposed in the literature to control such a complex behaviour. These strategies are meaningful from the biological point of view, and they aim to stabilize the dynamics of the system towards a positive equilibrium or at least to reduce the range of the fluctuations. In this framework, the following natural question arises: under which conditions such a stabilization or reduction of the range of fluctuation is global (that is, independent of the initial conditions)? In the last part of the talk, we will address such a question about the global dynamics of the controlled systems.

Some of the results presented in the talk have been made in collaboration with Frank Hilker (Univ. Osnabrueck, Germany), Juan Perán (UNED, Spain) and Hartmut Logemann (Univ. Bath, UK).

Keywords: Population dynamics, Chaos control, Global stability

Why fractional systems are not dynamical systems?*

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Based on the definition of dynamical system as a structure having semi-group property we claim that systems with fractional operators are not dynamical systems in this sense. We discuss possibility of another interpretation of fractional system in continuous and discrete cases.

Keywords: dynamical system, fractional derivative, semi-group property

A necessary condition of viability for fractional equations with the Caputo derivative

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In this talk viability results for nonlinear fractional differential equations with the Caputo derivative are proved. We give a necessary condition for fractional viability of a locally closed set with respect to a nonlinear function. An illustrative example is also contained.

Relative observability, duality for fractional differential-algebraic delay systems with jumps

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The problem of relative R-observability is considered for linear stationary fractional differential-algebraic delay system with jumps (FDADJ). FDADJ system consists of fractional differential equation in the Caputo Sense and an output equation. We introduce the determining equation systems and their properties. We derive solutions representations into series of their determining equation solutions and obtain effective parametric rank criteria for relative R-observability. A dual controllability result is also formulated.

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Keywords: Fractional differential equations, determining equations, differential-algebraic systems

A classification of linear controllable time-varying systems on time scales

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The problem of classification of linear time-varying systems with outputs

$$\begin{aligned} x^{\Delta}(t) &= A(t)x(t) + B(t)u(t), \quad && x(\cdot) \in \mathbb{R}^n,\ u(\cdot) \in \mathbb{R}^m \\ y(t) &= C(t)x(t), && y(\cdot) \in \mathbb{R}^p \end{aligned}$$

defined on a time scales $\mathbb{T}$ is considered. The sufficient and necessary conditions to guarantee the existence of a linear transformations of such systems to Brunovsky canonical form are given. The obtained results extend conditions given in [1] on any time model. Next, based on [2], these results are used to investigation of observer design for nonlinear system on time scale by considering a nonlinear system of the form

$$\begin{aligned} \widetilde{x}^{\Delta}(t) &= A\widetilde{x}(t) + \Phi(\widetilde{y}(t)), \quad \widetilde{x}(\cdot) \in \mathbb{R}^n \\ \widetilde{y}(t) &= C\widetilde{x}(t) \end{aligned}$$

where the output $\widetilde{y}(\cdot) \in \mathbb{R}$, smooth map $\Phi : \mathbb{R} \to \mathbb{R}^n$ and a pair (A, C) is in Brunovsky form.

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Nilpotency issues in the car+trailers' systems

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The kinematical model of a car towing an arbitrary number of passive trailers is classical and goes back to the beginning of the 1990s. For any fixed number of trailers it is a linear control system with two controls: the linear and angular velocity of the car. It is known (since the year 2000) that such a system is locally nilpotentizable in the sense of Sussmann. The nilpotency orders of the underlying real nilpotent Lie algebras (so-called Kumpera-Ruiz algebras) are effectively computed. However, it is not so with the dimensions of those algebras. Apart from the simplest cases, these

* The research was supported by Bialystok University of Technology grant No W/WM/4/2014.

dimensions have not yet been computed. Related with those are the nilpotent approximations (NA), at different points of the configuration space, of the underlying rank-two vector distribution modelling the possible evolutions of the kinematical system under consideration. The punch line in this subject is that, at many points of the configuration space, the NA's are NOT the Kumpera-Ruiz nilpotent algebras. Precisely this is the onset of a number of questions. Those algebro-geometrical questions, with all likelihood, deserve to be worked upon.

Polynomial accessibility condition of nonlinear control systems on homogeneous time scales⋆

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A necessary and sufficient accessibility condition for the set of nonlinear higher order input-output (i/o) delta differential equations is presented. The accessibility definition is based on the concept of an autonomous element that is specified to the multi-input multi-output systems. The condition is presented in terms of the greatest common left divisor of two left differential polynomial matrices associated with the system of the i/o delta-differential equations defined on a homogenous time scale which serves as a model of time and unify the continuous and discrete time. We associate the subspace $\mathcal{H}_\infty$ of the vector space of differential one-forms with the considered system. This subspace is invariant with respect to taking delta derivatives. The relation between $\mathcal{H}_\infty$ and the element of a left free module over the ring of left differential polynomials is presented. In the proof of the accessibility condition we use the fact that $\mathcal{H}_\infty$ is integrable and hence the existence of an autonomous element is shown. The presented accessibility condition provides a basis for system reduction, i.e. for finding the transfer equivalent minimal accessible representation of the set of the i/o equations which is a suitable starting point for constructing an observable and accessible state space realization. Moreover, the condition allows to check the transfer equivalence of nonlinear systems, defined on homogeneous time scales. Finally, illustrative examples that describe our results are presented.

Keywords: Nonlinear control systems, homogeneous time scale, left differential polynomials, accessibility condition

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Asymptotic properties of delayed matrix functions

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We study the asymptotic properties of delayed matrix functions, which are used for investigation of the linear differential system with constant matrix and constant delay, see [1]. Delayed matrix exponential is used for application of the step by step method and is defined as follows:

$$e_\tau^{Bt} := \sum_{j=0}^{k} B^j \frac{(t-(j-1)\tau)^j}{j!}$$

where $k = 0, 1, \dots$. The delayed matrix exponential e_τ^{Bt} is the matrix solution of the equation:

$$Y(t)' = BY(t-\tau). \tag{1}$$

Let matrices A, B commute and let A be a regular matrix, then the solution of the initial-value problem $y'(t) = Ay(t) + By(t-\tau), \quad t \in [-\tau, \infty), y(t) = \varphi(t)$, can be expressed for $t \in [-\tau, 0]$ as

$$y(t) = e^{A(t+\tau)} e_\tau^{B_1(t-\tau)} \varphi(-\tau) + \int_{-\tau}^{0} e^{A(t-\tau-s)} e_\tau^{B_1(t-\tau-s)} e^{A\tau} [\varphi'(s) - A\varphi(s)] ds$$

where $B_1 = e^{-A\tau}B$. Analogous results hold for the system of the linear differential equations of second order and delayed matrix function cosine and delayed matrix sine. The asymptotic properties of this is possible reduce to asymptotic properties of delayed exponential matrix due to relation:

$$\operatorname{Cos}_\tau \Omega \left(t - \frac{\tau}{2}\right) = \frac{e_{\frac{\tau}{2}}^{i\Omega t} + e_{\frac{\tau}{2}}^{-i\Omega t}}{2}, \qquad \operatorname{Sin}_\tau \Omega(t-\tau) = \frac{e_{\frac{\tau}{2}}^{i\Omega t} - e_{\frac{\tau}{2}}^{-i\Omega t}}{2i}. \tag{2}$$

which holds for any square matrix Ω.

Special delayed matrix functions are defined on intervals $(k-1)\tau \le t < k\tau$, $k = 0, 1, \dots$ (where τ is a positive delay) as matrix polynomials, continuous at nets $t = k\tau$. This fact complicates asymptotic analysis of delayed matrix functions. Therefore we study the sequence $\{e_\tau^{B(n\tau)}\}^\infty$ of values of delayed exponential of matrix at nets $n\tau$. In [3] is derived, that for constant matrix B which Jordan canonical form is diagonal matrix i.e. there is invertible matrix P such that $B = P^{-1} \operatorname{diag}(\dots \lambda_j \dots)P$ and for which modules of eigenvalues of B satisfies the inequality $e\lambda_j\tau < 1$ is this sequence approximately geometric progression and we find the constant matrix C such that the exponential of this matrix $e^{C\tau}$ is common ratio or more precisely

$$\lim_{n\to\infty} e_\tau^{Bn\tau} \left(e_\tau^{B(n+1)\tau}\right)^{-1} = e^{-C\tau} \tag{3}$$

Moreover is possible expressed this matrix $e^{-C\tau}$ in the form:

$$e^{-C\tau} = P^{-1} \operatorname{diag}\left(\frac{W_0(\lambda_1\tau)}{\lambda_1\tau}, \dots, \frac{W_0(\lambda_n\tau)}{\lambda_n\tau}\right) P,$$

where $W_0(x)$ is the principal branch of the well-known Lambert W-function (named after Johann Heinrich Lambert). This inverse function to the function $f(w) = we^w$. satisfying the relation $z = W(z)e^{W(z)}$ is a multi-valued (except at $z = 0$).

For real arguments $z = x$, $x > -1/e$ and real w ($w > -1$) the equation above defines a single-valued function $W_0(x)$. The Taylor series of $W_0(x)$ around 0 is given by

$$W_0(x) = \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} x^n = x - x^2 + \frac{3}{2}x^3 - \frac{8}{3}x^4 + \frac{125}{24}x^5 - \cdots,$$

which has radius r of convergence $r = 1/e$, for more details see [2]. As the function e^{Ct} is the matrix solution of the equation (1) the constant matrix C is matrix solution of the characteristic equation

$$C = Be^{-C\tau} \Rightarrow C = \frac{1}{\tau} P^{-1} \operatorname{diag}\left(W_0(\lambda_1\tau), \cdots, W_0(\lambda_n\tau)\right) P.$$

The relation for independent variable of the principal branch of Lambert W-function such that, depend variable has negative real part enables derive the theorem below.

Theorem 1. Let B is square matrix of order 2. We denote by $\operatorname{Tr}(B)$ the trace of matrix B and by $\det(B)$ the determinant of the matrix B and $D(B) = \operatorname{Tr}^2(B) - 4\det(B)$. Let the matrix satisfies inequality:

$$e\tau\sqrt{\operatorname{Tr}^2(B) + 2D(B)} < \sqrt{2}$$

and for $0 \geq D(B) < \frac{\pi}{2\tau}$

$$\sqrt{D(B)} - \frac{\pi}{\tau} < \operatorname{Tr}(B) < -\sqrt{D(B)}$$

and for $0 \geq D(B) < \frac{\pi}{2\tau}$

$$\tau\sqrt{\operatorname{Tr}^2(B) + D(B)} < -4\arctan\frac{\operatorname{Tr}(B)}{\sqrt{D(B)}}$$

then the norm of delayed exponential of matrix $|e_\tau^{Bt}|$ is bounded.

Theorem 2. Let B is square matrix of order 2 then the norm of delayed exponential of matrices $|\operatorname{Cos}_\tau \Omega\,(t)|$ and $\operatorname{Sin}_\tau \Omega(t-\tau)$ are not bounded.

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Quantum difference-differential equations

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Differential equations which contain the parameter of a scaling process are usually referred to by the name Quantum Difference-Differential Equations. Some of their applications to discrete models of the Schrdinger equation are presented and some of their rich, filigrane, and sometimes unexpected analytic structures are revealed.

A Lie-algebraic concept for obtaining basic adaptive discretizations is explored, generalizing the concept of deformed Heisenberg algebras by Julius Wess.

Some of the moment problems of the underlying basic difference equations are investigated. Applications to discrete Schrdinger theory are worked out and some spectral properties of the arising operators are presented, also in the case of Schrdinger operators with basic shift-potentials and in the case of ground state difference-differential operators. For the arising orthogonal function systems, the concept of inherited orthogonality is explained. The results in this talk are mainly related to a recent joint work with Sophia Rokopf and Lucia Birk.

An analogous situation on an equidistant lattice has been worked out.

Global observability of nonlinear systems

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Let us consider an analytic ordinary differential equation on a compact analytic manifold. It is defined by an analytic vector field on this manifold and forms a continuous-time dynamical system with the manifold being its state space. The system is equipped with an observation structure, which consists of an analytic observation map from the state space to a finite dimensional real affine space. Two states are called indistinguishable if the observation map gives the same values on the trajectories of the system starting from these states. The system is globally observable, if if does not admit distinct indistinguishable states. We develop necessary and sufficient conditions of global observability of an analytic system. They could be seen as extension of criteria of local observability that have been presented in [1]. The main object that is used to formulate the result is the observation algebra of the system. It is generated by the Lie derivatives of the components of the observation map with respect to the vector field that defines the system. Then a certain ideal of the ring of analytic functions on the product of the manifold with itself is constructed. The main result says that the system is globally observable if the real radical of this ideal is equal to some canonical ideal generated by the state space.

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Session: Decision Support in Negotiation

How do the decision makers decide? A multiple criteria decision making experiment

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Multi Criteria Decision Analysis (MCDA) offers a variety techniques that can support decision makers (DMs) in analyzing their preferences, building the ranking of alternatives or sorting them into the predefined categories [2,3,4]. These methods can differ in the formal construction as well the ways of the DM's preference elicitation. They may use the notion of a unique synthesizing criterion, the outranking relations, the direct assignments of rates or scores, the pair-wise comparisons expressed by linguistic scales, apply the concept of reference points, the fuzzy relations or verbal judgments [2,3,4]. Some guidelines are also proposed to help the DMs in selecting the most adequate MCDM method depending on decision situation and a number of research confirms the applicability of various MCDM methods for solving real-life problems [1,5].

In this paper, having analyzed the results of the questionnaire-based experiment, we investigate how the frames for the decision problem are defined and what concepts and notions are used by decision makers in analyzing the problem and formulating the argumentation lines to justify the decisions they make. In particular, we aim to find what are the conceptual categories and type of scales most frequently used by DM to describe their preferences; how precisely do they define a feasible decision space (e.g. frames, reference points) and what kind of the problems they experience during the decision process.

This experiment is a part of a bigger project on supporting multi-issue negotiations, in which the negotiators face the problem similar to the classic MCDM one. Therefore while analyzing the results we will refer to the negotiation situation and possible consequences that may affect the process of negotiation analysis.

Keywords: multiple criteria decision making, preference analysis, linguistic scales, reference points

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MARS approach in scoring negotiation offers for the verbally defined preferences of the negotiators

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1 Introduction

In this paper we consider the issue of evaluating the negotiation offers when the negotiator's preferences are verbally expressed. We present a new approach, called MARS (an acronym of Measuring Attractiveness near Reference Situations), which can be used to solve such decision-making problems. It combines the advantages of two methods, ZAPROS [7; 8] (an abbreviation of the Russian words: Closed Procedures near Reference Situations) and MACBETH [1; 2] (an acronym of Measuring Attractiveness by a Categorical Based Evaluation Technique), and makes it possible to determine the cardinal scores of the potential negotiation packages.

2 Justification

On the one hand, according to the study carried out by Larichev [6] the quantitative evaluations and comparisons of different objects are much more difficult for human beings than conducting the same operations using qualitative ordinal expressions of preference [8]. On the other hand, quantitative approach is frequently used in negotiation support since it allows building a negotiation offers scoring system [4; 9] as well as performing asymmetric and symmetric analyses of the negotiation process (measuring the scale of concessions, visualizing the negotiation progress, searching for the improvements in the contract negotiated by the parties, finding the arbitration solution of the negotiation problem, etc. [3; 5]). Taking that into account we have attempted to combine – in the form of MARS – qualitative and quantitative approaches to provide the negotiators with an aiding tool both easy to use and highly useful.

3 MARS

The method we propose exploits the idea of tradeoffs between the selected alternatives, enables negotiators to articulate their preferences verbally and results in complete ranking of the alternatives with scores measured on an interval scale. The MARS procedure consists of the following steps: 1. Determination of the evaluation scale for each criterion. 2. Pair-wise comparison of the differences in attractiveness between hypothetical alternatives, each with the best resolution level for all the criteria but one (the ZAPROS-like approach), and the ideal alternative (with the best evaluations for all the criteria), using the following semantic categories (the MACBETH-like approach): 'no', 'very weak', 'weak', 'moderate', 'strong', 'very strong' and 'extreme'. The comparisons are performed using M-MACBETH software, which automatically verifies their consistency and offers suggestions to resolve possible inconsistencies. 3. Solution of the linear program corresponding to the comparisons performed (using the MACBETH approach and M-MACBETH software) to obtain the scores from the 0-100 scale for the elements compared (the ZAPROS-like approach), i.e. to form the Joint Cardinal Scale (JCS). 4. Substituting the evaluations of the alternatives by the corresponding scores from the JCS, defining for each alternative the distance from the ideal alternative and ranking the alternatives according to the distance values in ascending order.

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Keywords: preference analysis, negotiation offer scoring system, verbal decision making, ZAPROS, MACBETH

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How quantum games can support negotiation?

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In negotiations there is always a difference in the level of knowledge of negotiators. They have no interest in disclosing their level of knowledge because it may be used against them by the other party. On the other hand, in integrative negotiations the full knowledge of the preferences of both parties can help in finding the optimal possible result. This dichotomy is common in negotiations and can only be resolved by mutual disclosure of both parties preferences. Alternatively they can use the third-party arbiter, who secretly collects information from both parties and then proposes optimal solution that is binding on the parties. In this paper we show that quantum entanglement can play the role of such an arbiter. We formulate negotiation as a quantum game where the players strategies correspond to unitary transformations of the given initial state in the Hilbert space. Quantum strategies are correlated through the mechanism of quantum entanglement and the result of the game is obtained by the collapse of the resulting transformed state. The range of strategies allowed for quantum players are richer than in a classical case and therefore the result of the game can be optimized. On the other hand the quantum game is completely save against eavesdropping and the players can be assured that this type of quantum arbitration is fair.

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Session: Difference and Differential Equations and Their Generalisation on any Time Scales

Boundedness and stability of discrete Volterra equations

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In many real-life situations, the present state and the manner in which it changes are both dependent on the past. An appropriate model for such situation are discrete Volterra equations, in which the present state depends on the whole previous history. So, Volterra difference equations are widely used for modeling processes in many fields. In our investigation we consider a Volterra difference equation of nonconvolution type in both: scalar and vector cases. We take into consideration this equation under different assumptions on its kernel. The sufficient conditions for existence of solution with required asymptotic properties are obtained. Particularly, we present sufficient conditions under which the considered equation has asymptotically constant solution, asymptotically periodic solution, weighed asymptotically periodic solution and bounded solutions.

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Comparison of boundedness of solutions of differential and difference equations

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In mathematics and computer science, an algorithm is a procedure (a finite set of well-defined instructions) for accomplishing some task which, given an initial state, will terminate in a defined end-state. Such algorithm usually is a recurrence relation. The difference equation, is an equation which defines a sequence recursively: each term of the sequence is defined as a function of the preceding terms. Most algorithms can be directly implemented by computer programs; any other algorithms can at least in theory be simulated by computer programs. In the this paper we are looking on the differences between asymptotic properties of solutions of the difference equation and its continuous analogies on example of third order linear homogeneous differential and difference equations with constant coefficients. We take the both equations with the same characteristic equation. We show that these equations (differential and difference) can have solutions with different properties concerning boundedness.

Keywords: differential equation, difference equation, recurrence, linear, third order, bounded solution
AMS Subject classification: 34A05, 34C11, 39A10

Boundedness of solutions of neutral type nonlinear difference system with deviating arguments

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In this paper, we consider three–dimensional nonlinear difference system with deviating arguments on the following form

$$\begin{cases} \Delta(x_n + p x_{n-\tau}) = a_n f(y_{n-l}) \\ \Delta y_n = b_n g(w_{n-m}), \\ \Delta w_n = \delta c_n h(x_{n-k}) \end{cases}$$

where the first equation of the the system is a neutral type difference equation. Firstly, the classification of nonoscillatory solutions of the considered system are presented. Next, we present the sufficient conditions for boundedness of a nonoscillatory solution. The obtained results are illustrated by example.

Keywords: difference equation, neutral type, nonlinear system, nonoscillatory, bounded, unbounded solution
AMS Subject classification 39A10, 39A11, 39A12

On discretization of polynomials corresponding to symmetric and antisymmetric functions in four variables

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The paper describes the symmetric and antisymmetric exponential functions of four variables, based on the permutation group S_4. We derive the explicit formulas for corresponding families of the orthogonal polynomials and some of their properties, namely the orthogonality, both the continuous one and the discrete one on the lattice. A general formula for orthogonal polynomials corresponding to symmetric exponential functions in n variables is given.

Keywords: alternating exponential functions, Fourier transform, orthogonal polynomials

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Session: Differential Operators: Algebra, Geometry, and Representations

Natural differential operators in the symmetric bundle

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On Dirac operators on Lie algebroids

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Dirac operators on Lie algebroids will be defined and investigated. The Dirac operator is determined by the Clifford multiplication and by the connection. The Clifford algebra structure comes from the metric structure in the fibers of the Lie algebroid and not necessarily in the fibres of the tangent bundle as it is in the classical case. The metric structure is just given by a symmetric tensor field that need not to be nondegenerate. The connections considered here are very general, not only as being of the algebroid type connections but also because we do not assume that they are torsion-free or compatible with the metric or with the Clifford structures. Consequently, we are able to consider the most general type of Dirac operators. Some particular cases will be discussed. One of them is the Clifford module of skew-symmetric forms on bundles equipped with a pseudo-Riemannian metric. Weitzenböck type formulas for the square of the Dirac operators will be derived.

Keywords: Clifford module, Dirac operator, Lie algebroid, covariant differential operators, Weitzenböck formula

Natural differential operators of symplectic manifolds

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Rummler in his paper [3] introduced the notation of vector-valued forms as tool for getting the Weitzebcoeck formula for exterior forms which are defined on Riemannian manifolds. He also introduced some new operators on exterior differential forms like j, alpha, grad, div.

I would like to present my result for the forms with values in a tangent bundle, which are defined on symplectic manifolds. In particular, I'll present some properties this operators, for example:

1. j is a antiderivation
2. grad is commute with the symplectic Hodge star
3. d is a composition alpha and grad
4. grad and -div are formally adjoint.

I would also like to say about the symplectic connections.

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Session: Fuzzy Calculus and Its Applications

Periodic fuzzy set on circular coordinates

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This paper presents circular polar coordinates transformation of periodic fuzzy membership function. The purpose is identification of domain of periodic membership functions in consequent part of IF-THEN rules.

1 Introduction

One of the most popular defuzzification methods is the center of gravity method. It computes the center of gravity of area under the membership function. However in the case of a periodic membership function, an area may be divided according to configuration of domain of them. For that reason, the interval which is the domain of membership function needs to shift for avoiding division of the area. The interval should have both ends where the value of membership function is equal to 0. The setting of the interval for each inference increases computational complexity. Therefore the authors propose the method of circular polar coordinates transformation of periodic membership and its defuzzification method in this study. The transformation to circular polar coordinates simplifies domain of periodic membership function.

2 Polar Coordinate Transformation of Periodic Fuzzy Membership function

In some fuzzy inference, time, season, direction, point of compass, and hue of color may be inferred. The membership functions of those fuzzy sets are periodic functions. The fuzzy grades of them return to the same value at regular intervals, and a closed interval is not confirmed. A membership function is said to be periodic with period $\omega > 0$, if we have $\mu(v) = \mu(v + \omega)$ for all variable v in carrier of the membership function.

Let v_0 be a fixed real number and let $\mu(v) : [v_0, v_0 + \omega] \to [0, 1]$ be a periodic membership function. The polar coordinates $\mu(v)$ (the radial coordinate) and θ (the angular coordinate) are defined in terms of Cartesian coordinates by

$$x = \mu(v)\cos\theta,\ y = \mu(v)\sin\theta, \quad \text{where} \ \ \theta = \frac{2\pi}{\omega}(v - v_0).$$

We can see easily that the transformation is unique for each membership function. Based on these conversions, we can prevent the domain of membership function from separating on the periodic interval.

3 Defuzzification Method

It is need that a singleton is calculated from a closed plane figure which is a periodic function on the circular polar coordinates. The center of gravity method is widely used as defuzzification method. However if the closed plane figure is continuous, the computation will be large. Therefore for the practice example to be simple and a significant reduction in computational complexity, we discretize the continuous interval and the membership function. Put $v_1 = v_0,\ v_2 = v_0 + \frac{\omega}{n-1},\ v_3 = v_0 + \frac{2\omega}{n-1},\ \cdots,\ v_n = v_0 + \omega$. Then we have $x_i = \mu(v_i)\cos\left\{\frac{2\pi}{\omega}(v_i - v_0)\right\},\ y_i = \mu(v_i)\sin\left\{\frac{2\pi}{\omega}(v_i - v_0)\right\}$, where $v_i \in [v_0, v_0 + \omega],\ i = 1, 2, \cdots, n$.

We approximate the closed plane figure to the polygon for simplification. From the points of the polygon (x_i, y_i) $(i = 1, 2, \cdots, n)$, the physical center of gravity is obtained as follows: $(x^*, y^*) = \left(\frac{1}{n}\sum_{i=1}^{n} x_i, \frac{1}{n}\sum_{i=1}^{n} y_i\right)$. By using the physical center of gravity, we propose the definition of a defuzzified value of the periodic function on the circular polar coordinates.

$$v^* = \begin{cases} v_0, & (x^* > 0,\ y^* = 0) \\ v_0 + \dfrac{\omega}{4}, & (x^* = 0,\ y^* > 0) \\ v_0 + \dfrac{\omega}{2}, & (x^* < 0,\ y^* = 0) \\ v_0 + \dfrac{3\omega}{4}, & (x^* = 0,\ y^* < 0) \\ v_0 + \dfrac{\omega}{2\pi}\arctan\dfrac{y^*}{x^*}, & (otherwise) \end{cases}$$

The argument of the physical center of gravity converted into the value on Cartesian coordinates which is domain of the primary periodic membership function $[v_0, v_0 + \omega]$.

The physical center of gravity is calculated as a defuzzification point in this paper. However there are some proposed method which defines geometric centroid and other one in a different meaning. In the future, other defuzzification method is discussed with development of this study. The distance of center of gravity from the origin of the coordinates system is considered as some indicator of approximate reasoning.

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Optimizing inventory management of a firm under fuzzy data

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The main objective of a good inventory management system is to keep the inventory costs to the minimum. There are several elements of inventory cost, such as ordering cost, transportation cost, frozen capital cost, cost of loss (i.e. aging), cost of lost sales due to inventory shortages, and others. Several inventory models have been built based on the above. There are two most commonly used inventory models: fixed order quantity system and replenishment system.

In the first system the quantity to be ordered is fixed and re-orders are made once the stock reaches a certain pre-determined level called safety stock. It means that the next order is typically fixed and based on the average consumption during the lead time plus some safety stock. Often in calculation the buffer stock is the one day inventory consumption.

Under the second system the quantity to be ordered is not fixed, the next order is decided based on the lead time of the material, maximum stock level, i.e. the ordered level changes with time.

In the paper we propose two tools of inventory management. One is based on the fixed order quantity model which takes into account several elements of inventory cost, as well as extra discounts. The tool deals with fuzzy concepts represented by Ordered Fuzzy Numbers. The second tool takes into account the dynamics and works on the base of replenishment system. This tool can be regarded as a kind of controller.

Dealing with the first tool the fuzzy optimization problem for the total cost function is formulated within the space, where all variables of the model are fuzzy. After the choice of a particular defuzzification functional an appropriate theorem is formulated which gives the solution of the fuzzy optimization problem.

Developing the second tool the authors face with situations when material demands are irregular in the production process. This result in no equal ordered levels as well as in different elapse times between orders.

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The application of ordered fuzzy numbers in the SAW procedure

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Multi-criteria decision making (MCDM) refers to making preferences (e.g. evaluation, prioritization, and selection) over the available alternatives that are characterized by the multiple criteria (Chen and Hwang 1992; Hwang and Yoon 1981). It is an important research topic, with extensive theoretical and practical backgrounds. When facing MCDM problems in real life, we have to take into account that the human judgments and preferences are often vague and complex, so decision maker cannot always appraise their preferences exactly. There are decision situations in which the information cannot be assessed precisely, but may be represented in a quality form, often by linguistic values.

The main purpose of this paper is to contribute the Ordered Fuzzy Numbers (OFN) for dealing with problem uncertain information given in linguistic terms which is difficult to express by expert when making decisions. We show that the Ordered Fuzzy Numbers allows us to take into account the ambiguities occurring at expression of linguistic variables. The concept of Ordered Fuzzy Numbers was introduced and developed by Kosiski and his two co-workers: Prokopowicz and Ślęzak in the series of papers (Kosiński, Prokopowicz, Ślęzak 2002a; Kosiński, Prokopowicz, Ślęzak 2002b; Kosiński 2006; Chwastyk, Kosiński 2013).

The new fuzzy simple additive weighting technique (FSAW) based on OFN is also presented. The evaluations of alternatives are given by linguistic expressions, where linguistic terms have to be quantified within previously determined value scale. We are focused on the problem of extending the value scale in evaluating criteria to take into account values between numerical ones, such us: "a more than 2" or "a less than 3", together with "2" and "3". To express such fuzzy preferences we used the ordered fuzzy trapezoidal numbers because they are easily to used and interpreted. An evaluation score in FSAW is calculated for each alternative by multiplying the scaled value expressed by ordered trapezoidal fuzzy number given to the alternative of that criterion with the weights of relative importance directly assigned by decision maker followed by summing of the products for all criteria. The total fuzzy scores of alternatives are next defuzzified and ranked by crisp value of those scores. The usability different techniques of defuzzification of the ordered fuzzy numbers from the viewpoint of their application to rank ordering alternatives obtained by FSAW were also discussed.

The proposed framework allows us to represent the information in a more direct and adequate way in case where we are unable to express it precisely by score from ordinal scale. This methodology handles the key aspects of decision making in a consistent and rational way. The presented method will be also supplied with numerical examples to illustrate the FSAW procedure.

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Session: Mathematics in Biology and Medicine

Effect of awareness programs with HAART in controlling the disease HIV/AIDS: a mathematical approach

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HIV/AIDS become a hazard infectious disease in both the developed and developing nations. It is one of the deadliest epidemics in human history. We cannot collapse the wideness of HIV/AIDS only by prescribing Anti-Retroviral Therapy (ART) or Highly Active Anti-Retroviral Therapy (HAART). People need to be enlightened about the disease HIV through awareness programs. Awareness programs cannot eradicate infections but help us in controlling the prevalence of the disease. In this research work, we consider a system consisting high-risk unaware individuals, HIV infected but not yet diagnosed individuals, diagnosed HIV-positive individuals who have not yet progressed to AIDS, those with clinical AIDS and aware susceptible individuals. We also consider the effect towards the population through media programs along with the drug therapy in our system. Our analytical and numerical results reveal that using awareness programs through media, comprising with drug therapy is more effective rather than the merely use of drug therapy to the infected individuals.

Effect of delay during transmission of the disease Cutaneous Leishmaniasis: a mathematical approach

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Cutaneous Leishmaniasis (CL) is a vector borne disease caused by a single-cell parasite and transmitted by female phlebotomous papatasi sand fly bites. CL is characterised by skin lesions. The disease starts as an erythematous papule at the site of the sand-fly bite on exposed parts of the body. People can carry some species of Leishmania asymptomatically for long periods and the reported incubation period for CL may be few weeks or several months. In this research article, a mathematical model of CL has been formulated considering the basic circumstances that susceptible human become infected through mass action after interaction with the infected vector. We also consider the fact that susceptible vector become infected after interaction with infected human. For better understanding of the disease CL, a time delay and its effect in the incubation period has been considered as there is a time interval between invasion by an infectious agent and appearance of the first sign or symptom of the disease. Theoretical analysis of our model shows that the effect of delay can changes the progression pattern of the growth of the disease. The numerical simulations also satisfy the effect of delay.

Criterions of multidimensional ν-similarity for birth-death processes

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We consider birth-death processes on the non-negative integers with a possible absorbing state at -1. A novel definition of multidimensional ν-similarity of two processes is introduced. The idea of multidimensional similarity is an extension of the ν-similarity concept introduced by Lenin et al. [1]. We give necessary and sufficient conditions of multidimensional ν-similarity of two birth-death processes in terms of their parameters.

Keywords: birth-death process, ν-similarity, multidimensional similarity

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Effects of environmental protection in population dynamics

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Due to the pressures of human activities, the natural environment is being degraded. Seeking the benefit of both the natural environment and humans, several conservation strategies have been proposed—and some of them implemented—to at least stop such a degradation. However, the impact of protection measures is not well understood, and for this reason they are occasionally criticized.

In this talk, we will consider environmental protection strategies of two different types: restrictions on human activity (harvesting biomass limits and no-take zones), and promotion of dispersion among isolated patches (ecological corridors). We will discuss how to take into account such strategies in classic discrete population models. Then, we will employ analytical and numerical tools to understand a little better the expected consequences of these protection measures.

Some of the results presented in the talk have been made in collaboration with Juan Perán (UNED, Spain) and Alfonso Ruiz-Herrera (Univ. Szeged, Hungary).

Keywords: Discrete population models, Metapopulations, Conservation strategies

Stability problem for the age-dependent predator-prey model

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In the classical mathematical ecology the best known model is Lotka-Volterra one. It describes the competition of two populations, predators and preys. This model is based on the system of ordinary differential equations. The classical Lotka-Volterra model assumes that each meeting of a predator with a prey finishes with eating of a prey. The natural modification of the model is considering of the age structure. We assume that the “chance” on eating prey by predator depends on predator’s age as well as prey’s one. The age-dependent model is based on the system of partial differential equations. We give criterions of stability for presented age-dependent predator-prey model.

Keywords: age-dependent model, von Foerster equation, Lotka - Volterra model, stability.

Families of ν-similar birth-death processes and limiting conditional distributions

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We study, for a given process $\mathcal{X}$, conditions under which ν-similarity is provided, relations between similarity and ν-similarity of processes and analyze (doubly) limiting conditional distributions. Conditions providing existence of ν-similar process to the given one with recursive formulas for birth and death rates are formulated. The appropriate example to illustrate formulas is given.

Keywords: birth-death processes, ν-similarity, orthogonal polynomials

Asymptotic properties of the von Foerster-Lasota equation and indices of Orlicz space

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We consider dynamical system induced by the von Foerster-Lasota equation. We study chaoticity of the system in the sens of Devaney and its strong stability in the Orlicz space generated by any φ-function. We present a description of the asymptotic properties considered semigroup in the context of the Matuszewska-Orlicz indices.

Keywords: von Foerster-Lasota equation, chaos, stability, indices of Orlicz space

Session: Quantitative Methods in Economics

Intuitionistic present value – an axiomatic approach

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The subject of considerations is the present value defined as the discounted utility of cash flow. We assume here also that: 1° the utility of current payment is equal to its nominal value; 2° the utility is odd function of payment nominal value. This axiomatic definition of present value is generalized to the case in which present value is equal to intuitionistic number. Additivity of present value, diversification rule and The First Gossen's Law are also taken into account.

Keywords: temporal utility, present value, intuitionistic number, the First Gossen's Law, diversification rule, additivity of present value.

The use of combined multicriteria method for the valuation of real estate

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A considered problem is valuation of real estate. The valuation is defined as the process of estimating e.g. market value of property. The market value is defined as the price most likely to be concluded by buyers and sellers of a property that is available for purchase [The International Valuation Standards, European Valuation Standards and EU directives – Trojanek 2010]. Valuation of property is made on the basis of information and transactions on the local market and therefore characteristics have a local nature. Unfortunately, on the local market are not always sufficient number of transactions that can be used for valuation. Particularly, the valuation always is based on the data of the similar properties. Solution of this problem can be a method of valuation of real estate based on a combination of two multicriteria decision-making methods i.e. the Analytic Hierarchy Process (AHP) and Goal Programming (GP). The mixed AHP and GP procedure for multicriteria real estate valuation has been designed especially for valuation in situation in which information are limited. The main objective of this technique is to extract the knowledge underlying the valuation process from specific characteristics. It could be used with intangible and scarce information. The Analytic Hierarchy Process (AHP) enable to incorporate the tangible aspect with the intangible by means of using paired comparisons in the valuation procedure. The Goal Programming (GP) enable to include both the scarce information available (objective) and the individual appraisers attitude with regards to the valuation process (subjective). The proposed method will be used for the valuation of the real estate located on Warsaw.

Comparison of the tails of market return distributions

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The aim of this study is to analyze the tails of the distributions of stock market returns and to compare the differences among them. It is a well-established fact that the vast majority of stock market return distributions exhibit fat tails (a bigger probability of extreme outcomes then in the case of the normal probability distribution). Apart from that there seem to be a popular opinion that most market returns are negatively skewed with a fatter left tail. There are many papers that examine the skewness of stock market returns [1, p.360], but not that many that focus directly on the comparisons between the tails of the distributions. [2] and [3] are two examples.

The issue of the tails of stock market return distributions is an important one as the tails and their probabilities represent the possible best and worst-case scenarios of any stock investment. In this study tails of the return distributions of mostly European stock market indices are analyzed and compared. The studied sample consists of 20 time series of daily logarithmic stock market returns from the period ranging from 1 January 2004 until 1 April 2014. The following indices were included in this study: WIG, WIG20, mWIG40 and MiS80 indices of the Polish Warsaw Stock Exchange; and S&P 500 (U.S.), Nikkei 225 (Japan), All Ordinaries Index (Australia), Bovespa Index (Brazil), SAX Index (Slovakia), PX Index (Czech Republic), OSE All Share Index (Norway), OMX Vilnius Index (Lithuania), OMX Tallinn Index (Estonia), OMX Riga Index (Latvia), OMX Helsinki Index (Finland), IBEX Index (Spain), FTSE 250 (United Kingdom), DAX Index (Germany), CAC40 (France), and BUX Index (Hungary).

The study utilizes two methods for comparing the tails of the distribution. A simple approached based on the sample kurtosis, with a small modification that allows for the calculation of kurtosis separately for both tales of the distribution and a more sophisticated approach based on the maximum likelihood fitting of the Generalized Pareto Distribution to both tales of standardized return distributions. The second approached is based on the assumptions of the Extreme Value Theory (EVT) and the Pickands-Balkema-de Haan theorem [4] and [5]. Both approaches provide similar conclusions. Results suggest that whether the left or the right tail of the return distribution is bigger varies from market to market. All four major equity indices of the Polish Warsaw Stock Exchange exhibited a fatter left tale. However in the whole sample it was actually more common for the right tail to be heavier, with 12 indices out of 20 exhibiting a fatter right tail then the left. The sample kurtosis indicated that all stock market return's distributions were heavy tailed, whereas the estimates of Generalized Pareto Distribution parameters did indicate standard or thin tails in two cases. The Jarque-Bera tests for all indices strongly indicate that the market return distributions are not normal. As for the in-sample skewness the majority of examined indices exhibited a positive skew with only five indices being negatively skewed.
JEL classification: G15, D81, C13

Keywords: Market returns; Extreme value theory; Fat tails

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Financing startups business – grants or preferential loans?

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Development of enterprises, especially SME (small middle enterprises) sector and micro-enterprises, is one of the main factor deciding about the financial situation of the country. According to the Central Statistical Office, the SME sector is responsible for producing about 47% of GDP (Gross Gomestic Product) in Poland and employs 6.2 million people. The number of active businesses in the SME sector is about 1.72 million and every year coming into being around 400 thousand of new businesses[1]. This number in compared to the years 2003–2005 is almost doubled. Direct cause

[1] data for the year 2011

can be found in a wider access to European Union funds under the operational programs such as: Regional Operational Programme financing from European Regional Development Fund (for example Regional Operational Programme of Podlaskie Voivodeship), Human Capital Programme (for example Action 6.2 “Support for and promotion of entrepreneurship and self-employment”), Innovative Economy Programme (for example Action 8.1 “Support for business activity in the field of electronic commerce”), Rural Development Programme (for example Action 312 “Creation and development of micro-enterprises”). It should be noted that the rate of ”survival” of the first year of the new company in Poland is currently around 70%. Particular attention should be paid to the issue of financing of new enterprises when financing of these come form with public money. In connection with the new prospect of funding from the European Union, we should consider whether the current system of financing such projects is effective and does not lead to distortions and fraud (intentional or due to lack of knowledge and “familiarity” in the conduct of business). At present, about 50% of newly established economic activities in the Podlaskie voivodeship financed by non-repayable grant is closed or suspended after the first year of operation. This ratio grows to about 70% after two years from the date of establishment. This volume is thus worse than the national average. Therefore, in my speech I make a comparison of the personal features and people business ideas (in example: age, seniority, education, investment value, etc.) to which is directed support in the form of non-repayable grants and people who decide to finance the start up their business using preferential loans. The study will apply selected methods of multi-criteria decision making.

Applications of line and surface integrals to economic issues

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In this talk we present applications of line and surface integrals to problem of calculation the total production and cash flow. Line (surface) integral of a scalar field may be used to express the value of total production at a fixed curve (surface) of expenditures for a given production function. On the other hand, line integral of a vector field expresses the resultant flow of money in transactions of purchase and sale. It takes into account the impact of the sales volume of one good for the price of another good. The last type – surface integrals of a vector field – may be used for calculations the amount of the total flow of a bulk material by any surface.

Session: Weak Partial Linear and Partial Chain Spaces and Their Geometry

On some configuration on finite non-desarguesian projective planes

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There are some configurations, which can characterize finite projective non-desarguesian plane. The aim of my talk is to present some questions connected with Fano configuration and the only 10_3 configuration B ([5]). The first Lenz's example of finite projective plane of order 16([3]) show that there are projective plane containing both quadrangles with collinear and non-collinear points. We present Fano configuration on Hughes, Figueroa and Hall planes of odd order. It is connected with extendability of projective plane to Minkowski plane. We show the existence of the configuration B in Figueroa planes of odd order. We expect that it is not embeddable in Hughes plane. The presentation is based on the material from common with K. Prażmowski article (in preparation).

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Projective realizability of Veronese spaces

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joint work with Krzysztof Prażmowski

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The notion of k-Veronesians (over semilinear spaces) was introduced by Naumowicz & Prażmowski in [3]. They generalize so called 2-Veronesians over a projective space considered by Tallini and Melone ([4], [1]).

Let $\mathfrak{D} = \langle S, \mathcal{L} \rangle$ be an incidence structure.

Definition 1 (comp. [3]) *For an integer $k \geq 1$ we define $\mathbf{V}_k(\mathfrak{D}) := \langle \mathfrak{y}_k(S), \mathcal{B} \rangle$, where $\mathfrak{y}_k(S)$ is the family of k-element multisets on S and the blocks $\mathcal{B}$ are the sets of type $\{a^{k-r}e \colon a \in L\}$ for $L \in \mathcal{L}$, $e \in \mathfrak{y}_r(S)$, and $0 \leq r < k$.*

Let distinct blocks of $\mathfrak{D}$ have at most λ points in common, then the same holds for $\mathbf{V}_k(\mathfrak{D})$. If $\mathfrak{D}$ is a $\left(\boldsymbol{v_r}\, \boldsymbol{b_\kappa}\right)$-configuration then $\mathbf{V}_k(\mathfrak{D})$ is a $\left(\binom{v+k-1}{k}_{k\boldsymbol{r}}\, \binom{v+k-1}{k-1}\boldsymbol{b}_{\boldsymbol{\kappa}}\right)$-configuration as well.

Fact 1 (comp. [3]) *Let $\mathfrak{M}$ be a semilinear space (i.e. an incidence structure in which any two distinct blocks intersect in at most one point). Then the following holds.*

(i) $\mathfrak{M} \cong \mathbf{V}_1(\mathfrak{M})$.
(ii) $\mathbf{V}_k(\mathfrak{M})$ *can be embedded in* $\mathbf{V}_{m\cdot k}(\mathfrak{M})$ *and* $\mathbf{V}_{k+m}(\mathfrak{M})$*, where m is a positive integer.*
(iii) $\mathfrak{M}$ *can be embedded in* $\mathbf{V}_k(\mathfrak{M})$.
(iv) If $\mathfrak{M}'$ *is a substructure of* $\mathfrak{M}$ *then* $\mathbf{V}_k(\mathfrak{M}')$ *can be embedded in* $\mathbf{V}_k(\mathfrak{M})$.

In essence, Fact 1 remains valid for arbitrary incidence structures.

Definition 2 (comp. [3]) *Combinatorial k-Veronesian is a structure $\mathbf{V}_k(S) = \mathbf{V}_k(\mathfrak{D})$, where $\mathfrak{D} = \langle S, \{S\}\rangle$. We write $\mathbf{V}_k(n) = \mathbf{V}_k(S)$ if $|S| = n$.*

Fact 2 ([3]) $\mathbf{V}_{k_1}(n_1)$ *can be embedded in* $\mathbf{V}_{k_2}(n_2)$ *if (and only if) $n_1 \le n_2$ and $k_1 \le k_2$.*

Fact 3 ([3]) $\mathbf{V}_3(3)$, $\mathbf{V}_2(n)$*, and* $\mathbf{V}_n(2)$ *are realizable on the real projective plane.*

Let $n \ge 3$ or $k \ge 3$. Then $\mathbf{V}_k(n)$ can be embedded in a Desarguesian projective space iff $n = 3 = k$ or $n = 2$ or $k = 2$.

If a Desarguesian projective space contains $\mathbf{V}_3(3)$ then the characteristic of its coordinate field is $\neq 2$.

Theorem 1 *Let $k \ge 3$ and $\mathfrak{M}$ be a semilinear space. If either $\mathfrak{M}$ contains a line on at least 4 points or $k \neq 3$ and $\mathfrak{M}$ contains a line with at least 3 points then $\mathbf{V}_k(\mathfrak{M})$ cannot be realized in a Desarguesian projective space.*

Theorem 2 *Let $\mathfrak{M}$ be a projective space and $k \ge 3$. There is no Desarguesian projective space in which $\mathbf{V}_k(\mathfrak{M})$ can be embedded.*

Theorem 3 *The Veronese space $\mathbf{V}_2(\mathfrak{P})$ where $\mathfrak{P}$ is the Fano plane cannot be realized in any projective space $PG(n, 2)$.*

Proposition 9 *Let $\mathfrak{V}$ be the Veblen configuration. The structure $\mathbf{V}_2(\mathfrak{V})$ can be realized in $PG(5, 2)$.*

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The projective line over finite associative ring with unity

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We discuss the projective line $P(R)$ over finite associative ring R with unity. It is defined as the set of free cyclic submodules $R(a;b)$. Obviously, every admissible pair (or unimodular in commutative or finite case) generates a free cyclic submodule. In certain cases free cyclic submodules may also be generated by non-admissible pairs. We give some examples of the projective line with unimodular and non-unimodular part. We describe the case of the projective line over local ring and we prove that this is sufficient to describe the projective line over finite commutative ring. We are also concerned with its automorphism group.

Projective line carries two non-trivial, mutually complementary relations: distant and parallel. We show that the distant graph on the projective line over finite ring is connected. Next we characterize the automorphism group of the distant graph on the projective line over an arbitrary local ring. In the case of rings Zp^n (p – prime) we explicitly count the cardinality of $P(Zn)$, $Aut(P(Zp^n))$, $PGL_2(Zp^n)$.

The characterization of the projective line over rings of order p^i, $i \leq 3$ is also known. In case p^3 we have exactly one noncommutative ring $T_2(GF(2))$ (the ring of triangular matrices over $GF(2)$) and the other are local. It is demonstrated that the description of projective lines over rings of prime power order suffices to characterize the projective line over finite ring.

Weak partial chain spaces and their products

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The theory begins, in my opinion, with the problem to characterize synthetic geometry of quadrics (or – even more generally – of algebraic varieties).

In my talk a *weak partial chain space* (WPCS) is an incidence structure (a block design) with three properties (cf. [1], [2]):

- through three distinct points there passes at most one block (*a chain*);
- each block is on at least four points;
- the structure is connected.

A WPCS is a *weak chain space* if, additionally, any triple of points pair wise joinable by a chain lies on a chain. Classic (and primary) model of such a structure is a quadric Q in a projective space with a family of conics (suitable plane sections) of Q distinguished. In this classic case another family of blocks appears natural: lines (linear *generators*) of Q, and some additional relations like the *tangency* are in use. What is more (another classic result): quadrics can be characterized both as chain spaces and (with few exceptions) as *polar spaces* i.e. as incidence structures with generators considered as the blocks.

Some standard operations on incidence structures are well known. In particular: the *Segre product* operation (cf. [4]), the *Veronese 'product'* (cf. [3]), and the *derived structure* (at a point). In the talk we discuss some introductory problems concerning foundations (and axiomatization) of the theory of weak partial chain spaces, with special emphasis imposed on these three operations. To be more exact: we firstly discuss which axioms (and notions) are 'preserved' under corresponding operations. In most cases only 'the most weak variants' are preserved! Another classic problem to characterize the automorphisms of products can be solved relatively easily. The third problem, a common one in case of projective quadrics, is read as follows: characterize the structure of chains via suitable structure of lines (and conversely). This one also can be solved for products of quadrics.

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On affinization of Segre products of partial linear spaces

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The term *affinization* is not widely used. Its idea however, is not only well known but also applied very often in geometry. It has been spotted in [3] and means construction of the complement of a hyperplane in some point-line space, inspired by construction of an affine space as a reduct of a projective space.

We characterize hyperplanes in the Segre product $\mathfrak{M}$ of partial linear spaces. These are similar to structures investigated in [1] and [2]. The general formula for hyperplane $\mathcal{H}$ in $\mathfrak{M}$ is determined. We also establish hyperplanes with properties, that are favorable to afinization procedure. The complement $\mathfrak{M} \setminus \mathcal{H}$ can be, in a natural way, equipped with the parallelism $\|_{\mathcal{H}}$ arising from the hyperplane $\mathcal{H}$. We prove that the relation $\|_{\mathcal{H}}$ is definable in terms of point-line incidence of the product $\mathfrak{M}$ like it is in most of geometries that resemble affine spaces. The automorphism group of $\mathfrak{M} \setminus \mathcal{H}$ is characterized. In particular, assumptions, under which the automorphisms of the complement are the restrictions of the automorphisms of the ambient space, are given. We are interested mainly in locally affine (i.e. covered by affine spaces) structures obtained as the result of affinization. We show that these are complements $\mathfrak{M} \setminus \mathcal{H}$, where $\mathfrak{M}$ is the product of Veblenian gamma spaces with lines thick enough.

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Affinization of Segre products of Grassmann spaces embeddable into projective spaces: applications and examples

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This talk is a continuation of the discussion *On affinization of Segre products of partial linear spaces*. We focus here on the Segre product $\mathfrak{M}$ which factors are Grassmann spaces embeddable into projective spaces with an intention to give some examples and show applications of the theory developed in the forementioned work.

For such $\mathfrak{M}$ we introduce a general construction of a hyperplane, which idea is based on the characterization of hyperplanes in Grassmann spaces provided in [4] (see also [1], [2], [3]). The key ingredient here is a multilinear form, or actually, a segment-wise semilinear and alternating form μ. The hyperplane $\mathcal{H}(\mu)$ is formed by those points of $\mathfrak{M}$ that are killed by μ. As a specific case of $\mathfrak{M}$ the product of projective spaces can be taken, and if $\mathfrak{M}$ is equipped with additional reflexive bilinear forms, the product of polar spaces or polar Grassmann spaces or a mixture of those can also be taken. Under some assumptions on μ we show that non-degenerate and flappy hyperplanes in $\mathfrak{M}$ do exist.

Hyperplanes in the Segre product of two projective spaces are characterized. The complete characterization of hyperplanes in the general case is challenging and worth realization, but it is not our goal at the moment. One of the reasons is that such characterization involves multilinear algebra and many results in this area can be found in the literature or the Internet, but none of them gives an ultimate answer to our problems.

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Student Session

Being an academic researcher: what's it like?

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Many gifted young women and men facing the choice of a career, ask themselves what it is like to be an academic researcher. In this talk, I will offer a personal perspective on what working in academia is for me and why I love what I do. I will discuss the fundamental mechanisms of the scientific enterprise, such as publications, peer review, and allocation of research funds, and outline three fundamental responsibilities of an academic: research, teaching, and service. I will offer some thoughts on the daily life of a scientist and the difficult problem of career planning. Finally, I will briefly discuss the skills that one needs to master in order to be successful as a scientist. The lecture is intended primarily for students who are considering a career in academia, although alternative perspectives on this topic from other researchers are warmly welcome.

Circulant matrices and their application to solving polynomial equations

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Like some mathematical topics, circulant matrices can be compared to fresh buns someone has just taken out of the oven, which just cry out to be worshipped and to wonder about their unusual properties. The aim of the talk is to provide basic definitions and key theorems used for solving polynomial equations of degree ≤ 4.

We fix hereafter a positive integer $n \geq 2$. We will focus especially on the n-dimensional complex vector space $\mathbb{C}^n$ and the ring of $n \times n$ complex matrices $\mathbb{M}_n$. Let $T : \mathbb{C}^n \to \mathbb{C}^n$ be a *shift* operator as defined:

$$T(v_0, v_1, \ldots, v_{n-1}) = (v_{n-1}, v_0, \ldots, v_{n-2}) \ . \tag{1}$$

Definition 1. The circulant matrix $V = \operatorname{circ} v \in \mathbb{M}_n$ *associated to* the vector $v \in \mathbb{C}^n$ is the matrix whose rows are given by iterations of the shift operator acting on v; its k^{th} row is $T^{k-1}v$, $k = 1, 2, \ldots, n$:

$$V = \begin{bmatrix} v \\ Tv \\ \vdots \\ T^{n-1}v \end{bmatrix} = \begin{bmatrix} v_0 & v_1 & \cdots & v_{n-1} \\ v_{n-1} & v_0 & \cdots & v_{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ v_1 & v_2 & \cdots & v_0 \end{bmatrix} \ . \tag{2}$$

We denote by $\operatorname{Circ}(n) \subset \mathbb{M}_n$ the set of all $n \times n$ complex circulant matrices.

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How to create effective admissions system?

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I describe the problem faced by admission system, which allows assignment specified number of people to institutions, like colleges.

Solution of the admission problem is based on the algorithm created by D. Gale and L. Shapley.

Starting of the marriage problem and the college admission system, which have regard the individual preferences of applicants and institutions, we will try understand, how this algorithm works and we show benefits it brings.

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The time scales calculus

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The theory of time scales is a relatively new branch of mathematics. It plays a special role in engineering practice, for example in control theory when we consider controllability, observability and realizability of dynamic systems. The idea of the time scales is the unification of the results obtained for continuous and discrete dynamical systems.

In this talk we will present basic information about calculus on time scales. We will present delta derivative, the Riemann integral, improper integrals and the chain rule on time scales. Moreover, these issues will be compared on different time scales. Presented theory will be ilustrated by examples and practical applications.

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The best card trick – power of permutations

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Let D be an ordinary deck of cards, M a powerful magician, A a smart magician's assistant and P an auditorium. P chooses five cards from the set D and A chooses one card from these five – the special card H. Next A hides H and puts the rest of chosen cards on the table. M enters to the presentation and guesses H knowing only the order of these four cards. We show how M can use properties of permutations to guess H not only when $|D| = 52$ but also in situations where we have for example 124 cards in D or if the number of chosen cards is different five. Moreover we present some facts about coding information and point out the role that some mathematical subjects (like modular arithmetic, lexicographical order of permutations, the pigeonhole principle and some concepts from number theory) play in these card tricks. In result we show that card tricks can be used to review and teach some elements of discrete mathematics.

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Riesz spaces, Riesz homomorphisms, and the Banach-Stone theorem

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For a topological space X we define $\mathcal{C}(X, \mathbb{R})$ to be the $\mathbb{R}$-algebra of all continuous functions $f : X \longrightarrow \mathbb{R}$. It is easy to see that if topological spaces X and Y are homeomorphic, then the algebra $\mathcal{C}(X, \mathbb{R})$ is isomorphic to $\mathcal{C}(Y, \mathbb{R})$.

In the talk, we will discuss the notions of Riesz space and Riesz homomorphism, and explain how to use them to prove the following version of the Banach-Stone theorem.

Theorem 1. *Let X and Y be compact Hausdorff spaces. Suppose that the algebras $\mathcal{C}(X, \mathbb{R})$ and $\mathcal{C}(Y, \mathbb{R})$ are isomorphic. Then X is homeomorphic to Y.*

The talk will be based mainly on [2].

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Around Banach-Tarski paradox

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The Banach-Tarski paradox is a theorem in set-theoretic geometry, which states, in layman's terms, that a solid ball can be decomposed into a finite number of pieces that by rotating and translating can be put back together to yield two identical copies of the original ball. This theorem was published in 1924 in the paper [1]. The reason the Banach-Tarski theorem is called a paradox is that it contradicts basic geometric intuition.

To state the Banach-Tarski theorem precisely, and put it in a more general context, we need some notions. We will write $A = A_1 \,\dot\cup\, A_2 \,\dot\cup \cdots \dot\cup\, A_n$ or $A = \dot\bigcup_{i=1}^{n} A_i$ to denote that a set A is a disjoint union of sets $A_1, A_2, \ldots, A_n$. Suppose a group G acts on a set X. We will say that subsets A, B of X are *G-equidecomposable*, written $A \cong B$, if there exist $A_1, A_2, \ldots, A_n \subseteq X$ and $f_1, f_2, \ldots, f_n \in G$ such that

$$A = \dot\bigcup_{i=1}^{n} A_i \quad \text{and} \quad B = \dot\bigcup_{i=1}^{n} f_i(A_i).$$

A subset E of X is called a *G-paradoxical set* if there exist A, B such that

$$E = A \,\dot\cup\, B, \quad A \cong E, \quad \text{and } B \cong E$$

(figuratively speaking, the set E can be decomposed into finitely many pieces from which two copies of E can be put together using elements of G). A group G is called *paradoxical* if G is a G-paradoxical set when G acts on itself via multiplication from the left.

In the above terminology, the Banach-Tarski theorem asserts that the solid ball B in the space $\mathbb{R}^3$ is G-paradoxical, where G is the group of rigid motions in $\mathbb{R}^3$ (generated by rotations and translations). In the talk we will present a proof of the Banach-Tarski theorem, based on the existing literature on the topic (e.g. [2], [4], [5]). Key steps in the proof are the observations that the group SO_3 of rotations of $\mathbb{R}^3$ contains a free group F_2 with two generators, and that F_2 is a paradoxical group. From this it follows by the use of the Axiom of Choice that there exists a countable subset D of the sphere S^2 such that $S^2 \setminus D$ is SO_3-paradoxical; this result is known as the Hausdorff Paradox and it plays an important role in the proof.

In this talk we will also present some generalizations of the Banach-Tarski theorem and we will discuss its relationship with amenable groups ([3]).

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Curves and splines in computer graphics

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In computer graphics two basic primitives can be distinguished – a line segment and a polygon. These two are used to represent various objects. The problem arises when there is a need to construct complex shapes consisting of curves and smooth surfaces. Though mentioned means are enough to approximate any curved structures, this results in rapid increase of the stored data. What is more, such a technique rarely provides sufficient accuracy. Therefore it is nothing unusual that graphics software is supplied with the bigger set of basic shapes like arcs, circles, cubes, spheres, etc.

However, when it comes to modelling irregular shapes, the approach is quite different and uses primitives that can be irregularly curved. To achieve that, graphics systems implement methods to define a curve based either on the interpolation or the approximation of control points [1][2]. Such a curve is splitted into number of pieces called segments in order to preserve flexibility and generality. The point at which neighbouring segments meet is called a joint. Since curve is divided into parts, it is essential to qualify how these sections can be connected.

Another fundamental step is to draw obtained curve. To do so in a short time, which is a key factor in a real-time computer graphics, the effort to determine the coordinates of each point of the graph should be minimized [3]. Among explicit, implicit and parametric description of a curve, the latter satisfies placed condition.

During the speech raised issues will be considered in more detail.

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Algebraic proof of the fundamental theorem of algebra

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The fundamental theorem of algebra was proved by Gauss in 1799 in his Ph.D. thesis. Since then many other proofs of the theorem have been published (the book [2] is a summary of the known proofs). Existing proofs can be divided into three categories: these based on topological considerations, the analitycal proofs, and the algebraic ones. The proofs from the last category use only algebraic methods, plus the fact that every odd-degree polynomial with real coefficients has a real root and that every complex number has a square root.

In this talk we will present Derksen's proof of the fundamental theorem of algebra, published in [1], which is related only to linear algebra. The proof is based on the observation that the fundamental theorem of algebra is equivalent to the statement that every square matrix with complex coefficients has an eigenvector. It uses the structures of real symmetric matrices and their complex extension - Hermitian matrices.

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On distributive rings and modules

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A module M is distributive if the lattice of submodules of M is distributive, i.e. $A \cap (B + C) = (A \cap B) + (A \cap C)$ for all submodules A, B, C of M. A ring R is said to be right (resp. left) distributive if R is distributive as a right (resp. left) module over itself. Among commutative integral domains, distributive rings are precisely Prüfer domains, and noetherian distributive rings are precisely Dedekind domains.

In this talk some examples, characterizations and properties of distributive rings and modules will be presented. In particular, it will be shown that the Köthe conjecture has an affirmative answer for the class of right distributive rings. Localizations of right distributive rings will also be discussed.

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Conley index and its applications

Daniel Strzelecki

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The most famous topological invariants used in nonlinear analysis are the Brouwer degree, the Morse and Conley index theories. The aim of my talk is to sketch definition and some properties of Conley index theory.

1 Construction

Let $f \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R}^n)$ and let $\varphi : \Omega \to \mathbb{R}^n$ be a flow generated by the differential equation:

$$\dot{y}(t) = f(y(t)), \tag{1}$$

where $\Omega \subset \mathbb{R}^n \times \mathbb{R}$ is an open subset such that $\mathbb{R}^n \times \{0\} \subset \Omega$. In other words: $t \mapsto \varphi(x,t)$ is the solution of the Cauchy problem: $\begin{cases} \dot{y}(t) = f(y(t)) \\ y(0) = x \end{cases}$

Definition 1. *We say that a set $A \subset \mathbb{R}^n$ is* invariant set *of the local flow φ if A is a union of solution curves of equation* (1).
If an invariant set A is maximal in some its neighbourhood (named isolating block*) then we say, that A is* an isolated invariant set.

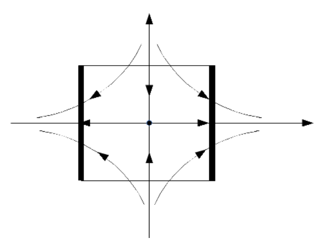


Fig. 1. Stationary point is the isolated invariant set.

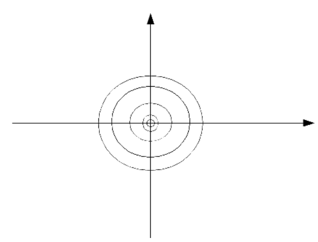


Fig. 2. There is no isolated invariant set.

Definition 2. *For the isolated φ-invariant set S we say, that a pair of compact spaces (N, L) where $L \subset N$ is* index pair *for S if:*

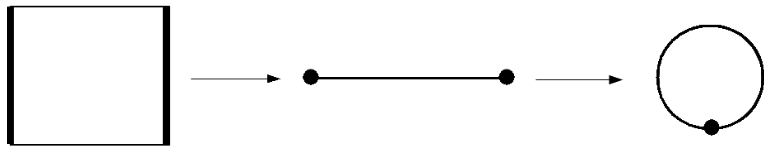
1. *$\overline{N \setminus L}$ is an isolating block for S,*
2. *L is a positive invariant set, i.e. if $x \in L$ and $\varphi[[0,t],x] \subset N$ then $\varphi[[0,t],x] \subset L$,*
3. *L is the set of rest points for N, i.e. if $x \in N$ and for some $t_1 > 0$ we have $\varphi(t_1, x) \notin N$ then exists $t_0 \in [0, t_1]$ such that $\varphi(t_0, x) \in L$.*

Example 1. Consider an equation like in Figure 1 $\dot{x} = Ax$ where $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Let $S = \{(0,0)\}$ be the isolated invariant set and we can take $N = [-1,1] \times \{-1,1\} \cup \{-1,1\} \times [-1,1]$ and $L = \{-1,1\} \times [-1,1]$ - bordered in Figure 1. It is easy to check that the pair (N, L) satisfies the definiton of index pair.

If we find good index pair for the isolated invariant set S then we can in simple way define Conley index of S.

Definition 3. Conley index for an φ-invariant set S *is a homotopic type of the pointed space* $(N/L, [L])$ *where* (N, L) *is the index pair for* S.

Example 2. Conley index homeomorphic to S^1 with the distinguished point for the isolated invariant set in Figure 1.



Remark 1. Definition of Conley index is correct because for any isolated invariant set there exists an index pair and the definition is independent of choosing an index pair. Formally: Conley index is index of an isolating block, but we can say that this is index of an invariant set (with a fixed flow).

2 Properties and applications

Theorem 1. *1. (Wazewski properties) If N is the isolating block for S with nontivial Conley index (not contractible to point) then $S \neq \emptyset$ i.e. in int(N) exists a nonempty invariant set.*

2. (Continuation) If N is the isolating block for all flows φ_λ, $\lambda \in [\lambda_0, \lambda_1]$ then Conley index of N is the same with flows λ_1 and λ_2.

With Theorem 1 Conley index is very useful in bifurcation theory. If we want to solve parametrized equation $\nabla_x g(\lambda, x) = 0$, where $g : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$, then we can consider a family of flows generated by differential equations $\dot{x} = \nabla_x g(\lambda, x)$. Solutions of the first equation are stationary points (so invariant sets also) of that flows.

Suppose that we know some family of solutions (for example: for all λ holds $\nabla_x g(\lambda, 0) = 0$). If we can compute Conley index of isolated invariant sets in some λ_1 and λ_2 and if we can say that these indices are different (this is no obvious in general situation) then we know that some $\lambda \in [\lambda_1, \lambda_2]$ is the bifurcation point of new (nontrivial) solutions.

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The axiom of choice in algebra

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The axiom of choice states that for every indexed family $(S_i)_{i \in I}$ of nonempty sets there exists an indexed family $(x_i)_{i \in I}$ of elements such that $x_i \in S_i$ for every $i \in I$. There are many statements that, in ZermeloFraenkel set theory, are equivalent to the axiom of choice. The most important among them are the Kuratowski-Zorn lemma (which says that every non-empty partially ordered set in which every chain has an upper bound contains at least one maximal element) and the well-ordering theorem (which says that every set can be well-ordered).

The theme of this talk will be examples of using the axiom of choice or its equivalences in proving significant algebraic statements. For instance, the proof of the existence of a basis for any nonzero vector space uses the Kuratowski-Zorn lemma, and in fact, in Zermelo–Fraenkel set theory, the assertion that every vector space has a basis is equivalent to the axiom of choice ([3]). Another algebraic equivalent of the axiom of choice is the statement that every nonempty set can be endowed with a cancellative semigroup structure ([2]). Among other major algebraic results whose proofs use the axiom of choice are the following theorems:

1. Every field has an algebraic closure.
2. Every commutative unital ring contains a maximal ideal.
3. The prime radical of a commutative ring coincides with the set of nilpotent elements of the ring.
4. If every prime ideal in a ring is finitely generated then every ideal in this ring is finitely generated.
5. Every torsion-free abelian group can be totally ordered.

In this talk we will show how the axiom of choice can be applied to prove these results.

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Stability of a partially damped rotating Timoshenko beam

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We consider a model from [1] of the rotation of a Timoshenko beam in horizontal plane whose left end is rigidly clamped into the disk of a driving motor. We consider the problem of stability of the system after introducing a particular type of damping. Let r be the radius of the disk and let $\theta = \theta(t)$ be the rotation angle as a function of the time $t \geq 0$. The beam is controlled by angular acceleration of the disk, $u(t) := \ddot{\theta}(t)$. If $w(x,t)$ denotes the deflection of the center line of the beam at the location $x \in [0,1]$ (the length of beam is assumed to be 1) and the time $t \geq 0$, and $\xi(x,t)$ – the rotation angle of the cross section area at x and t and if we assume the rotation to be slow, w and ξ are governed by two dimensionless partial differential equations

$$\begin{cases} \ddot{w}(x,t) - \mu^2 \dot{w}''(x,t) - w''(x,t) - \xi'(x,t) = -u(t)(r+x) \\ \ddot{\xi}(x,t) - \xi''(x,t) + w'(x,t) + \xi(x,t) \quad = u(t) \end{cases} \tag{1}$$

for $x \in (0,1)$ and $t > 0$, where μ is a damping constant, with boundary conditions

$$\begin{cases} w(0,t) = \xi(0,t) = 0 \\ w'(1,t) + \xi(1,t) = \xi'(1,t) = 0 \end{cases}$$

and initial conditions

$$\begin{cases} w(x,0) = \dot{w}(x,0) = \xi(x,0) = \dot{\xi}(x,0) = 0 \\ u(0) = 0 \end{cases}$$

Definition 1. *If a system maps every input u in $L_2(0,\infty)$ to an output y in $L_2(0,\infty)$ and*

$$\sup_{u \neq 0} \frac{||y||_2}{||u||_2} < \infty,$$

the system is stable.

We will consider control u as our input, and behavior of beam at $x = 1$ as our output; it means that our observation is any nontrivial linear combination of $aw(1,s) + b\xi(1,s) + cw'(1,s) + d\xi'(1,s); a,b,c,d \in \mathbb{R}$. To check stability we use a well-known theorem, here stated in a formulation from [2].

Theorem 1. *A linear system is stable if and only if its transfer function G belongs to*

$$H_\infty = \{G : \mathbb{C}_0^+ \to \mathbb{C} | G \textit{ analytic } \& \sup_{Re\ s>0} |G(s)| < \infty\}$$

with the norm $||G||_\infty = \sup_{Re\ s>0} |G(s)|$. In this case, we say that G is a stable transfer function.

We consider our equation in the operator form

$$\begin{pmatrix} \ddot{w} \\ \ddot{\xi} \end{pmatrix} + B \begin{pmatrix} \dot{w} \\ \dot{\xi} \end{pmatrix} + A \begin{pmatrix} w \\ \xi \end{pmatrix} = \begin{pmatrix} -r - x \\ 1 \end{pmatrix} u(t),$$

where $A : D(A) \to L^2((0,1), \mathbb{R}^2)$ is a linear operator defined by

$$A \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} -y'' - z' \\ -z'' + y' + z \end{pmatrix}$$

for $\begin{pmatrix} y \\ z \end{pmatrix} \in D(A) = \left\{ \begin{pmatrix} y \\ z \end{pmatrix} \in H^2((0,1), \mathbb{R}^2) \middle| \begin{matrix} y(0) = z(0) = 0 \\ y'(1) + z(1) = z'(1) = 0 \end{matrix} \right\}$; and $B : D(B) \to L^2((0,1), \mathbb{R}^2)$ is a symmetric linear damping operator with $D(B) = D(A)$ defined by

$$B \begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} -\mu^2 y'' \\ 0 \end{pmatrix}.$$

We analyze stability of Timoshenko beam in 3 situations, depending on the value of damping constant: $\mu = 0$, $\mu > 0$ and limit case $\mu = \infty$. We will show that in all cases our system (1) is unstable. All eigenvalues of undamped Timoshenko beam are on the imaginary axis – constant total energy – whereas the considered type of damping moves half of the spectrum to the negative part of the real axis, causing heat-type dissipation of energy. The other half is disturbed, but stays on the imaginary axis, which means that the energy of the part of the system never vanishes – the system in question is unstable.

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Tomographic image reconstruction methods

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X-ray computed tomography is a worldwide used technology of medical imaging. Providing an ability to visualize anatomical structures in form of 2D or 3D images, it plays a tremendous role in identifying missed threats to patients and confirming diagnoses posed by traditional diagnostic techniques. The aim of this talk is to present mathematical basics of computed tomography and show an overview and comparison of 2D image reconstruction algorithms used after the data collection provided by X-ray imaging.

Mathematical background was created by Austrian Johann Radon in 1917, who proved that it is possible to reconstruct 2D or 3D object from the infinite number of its projections. The result of each projection is specified by a Radon transform (called a sinogram), an integral transform over the straight lines, describing radiation attenuation coefficients of examined object [1].

We can divide reconstruction algorithms into two main groups: analytical and iterative methods. Development of representatives of the first group focuses on creating accurate and efficient methods of inverting sinograms in order to yield an original distribution of attenuation coefficients. They mainly use the Fourier transform to solve the problem, basing on the projection-slice theorem [2]. The group of iterative methods splits into classes of algebraic algorithms, focused on defining and solving equations describing cross sections [2], and statistical methods, using models based on structural and operational specifics of concrete scanners [3].

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