Mizar in Isabelle for Formal Abstracts*

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Abstract

One of the main goals of the Mizar project has been to create a formal system that would be attractive for mathematicians. Various developed features have therefore became an inspiration for extensions and improvements in other systems. At the same time, the architecture of Mizar has not been flexible enough to accommodate the solutions developed in other systems. In this talk we present a combination of the Isabelle – a modern logical framework – with a Mizar object logic and argue that it can serve as an attractive environment for formal mathematics. Indeed, the Mizar foundations are a variant of set theory, which is familiar for mathematicians. Its proof style does correspond to natural proofs. And the type system was designed to correspond to how mathematicians classify objects. We will finally discuss the various mechanisms that allow for grater usability for mathematical statements and proofs.

The Mizar project [1] from its beginning aimed to make a system for human readable formalization of mathematics. Many aspects distinguish it from other proof assistants. Its proof style imitates informal mathematical proofs. Its type system tries to express how mathematicians use and categorize mathematical objects. Combining these two features provide a more intuitive environment for formalized mathematics than other systems [8]. Furthermore, formalized Mizar results have been gathered in the *Mizar Mathematical Library* (MML). Its focus is on mathematical results, which makes it complementary to the libraries of various other proof assistants, including results that have not been formalized in other systems, such as the theory of lattices, topological manifolds, and random access Turing machines.

In this talk we will present a combination of a modern logical framework – Isabelle – with the foundations and the proof style of Mizar and discuss the usability of the resulting proof environment for the development of formal mathematical abstracts. Our Mizar emulator [7] created in the Isabelle logical framework provides selected constructs from the Mizar language [5]. We imitate the type system including intersection types and structures [6], as well as higher-order concepts, such as set comprehensions and schemes [4].

We will argue that the environment is more convenient for stating and formalizing formal mathematical statements. It is possible to naturally state mathematical object classifications, for example: `X` is `n`-dimensional topological-manifold is a compact Mizar-like statement that is clear to a mathematician, but stating it in many systems would require unnatural constructions. We show that it will be possible import and cross-verify the whole MML with all its mathematical results. We will present various manually re-formalized Mizar theorems including results from set theory, algebra and random access Turing machines. We will also discuss the Mizar level of human readability.

We discuss our foundations of the Mizar system as an object logic in the Isabelle logical framework, especially the number of needed constants and axioms. Then we focus on faithful

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imitation the Mizar definitional mechanisms. We show adequate mechanisms for each kind of these definition including the Mizar structures that allows multiple inhabitance. Finally we show also an experimental mechanism that provides selected Mizar type information in justifications of proof steps.

Various extensions of other proof assistants that imitate the Mizar language have been proposed. Examples include the Mizar Mode for HOL [11] and the Isar language for Isabelle [10]. These are however limited to a few rules the Jaśkowski natural deduction style [3], and omit other crucial parts of mathematical text, such as Mizar definitions of more complex objects that require Mizar-style justification of correctness. Similarly, there have been a number of attempts to translate the Mizar logic to various other formalisms. Urban [9] exported the MML to the TPTP first-order language, and with Brown this was extended to higher-order logic [2]. Such approaches try to preserve the semantics of Mizar, however do not preserve any of the user commands or notations. In consequence such translations significantly reduce proof readability, so important for formal proof abstracts.

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