

# Library integrity and redundancy (MIZAR )

Adam Naumowicz

Institute of Computer Science  
University of Białystok, Poland

[adamn@mizar.org](mailto:adamn@mizar.org)

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# Notable milestones from the MIZAR library's early history

- 1973 - MIZAR first presented by Andrzej Trybulec in a seminar at Warsaw University
- 1989 - MML (MIZAR Mathematical Library)
- 1990 - FM (Formalized Mathematics)
- 1995 - JFM (Journal of Formalized Mathematics)
- 1996 - CCL (MIZAR formalization of '*Compendium of Continuous Lattices*')



# MML foundation

- Before 1989: Central Archive of Mizar Texts coming with Mizar-4
  - basic set theory, topology, field algebra
  - more advanced formalizations like that of K. Borsuk's article on the homotopy types of some decomposition spaces
  - overlapping set theoretical preliminaries
  - functions and relations were disjoint
  - various geometries in different formal frameworks
  - groups, rings, fields, and vector spaces were all distinct, without any intrinsic hierarchy
- Main MIZAR Library (only later renamed to MIZAR Mathematical Library, initially multiple repositories were supposed to be created!)
- The Library Committee of the Association of Mizar Users was officially established to collect Mizar articles and to organize MML



# Current library's statistics

Current MML includes:

- 3300K LOC
- 1412 'articles'
- 65K theorems
- 13K definitions

More statistics on Grzegorz Bancerek's MMLQuery website  
([mmlquery.mizar.org](http://mmlquery.mizar.org))



# Mizar without (full) MML

Local libraries are sometimes created for:

- Work in progress projects
  - unstable terminology
  - known gaps (left for future or treated as axioms)
- Didactic purposes
  - simplified searching (e.g. all references in one resource file)
  - extra or non-standard axioms
  - turning off some automation



# Maintaining centralized library integrity

- General meaning:
  - organizing all basic definitions and derived theorems available to the users formalizing new theories on top of them
  - avoiding repetition and generalizing available data
- Practical tasks:
  - proper choice of symbols and notation for new (and old) constructors
  - maintaining a uniform hierarchy of standard types and structures defined in the library
  - detecting redundant (repeated or weaker) theorems
  - fixing cumbersome formulation of statements



# Some library integrity challenges

- Should various axiomatizations be allowed or not (and if only one, then which one should be chosen)?
  - looking for the most useful number sets hierarchy
  - different MML approaches to lattices, category theory, graph theory etc.
- Should we lean towards more abstract or concrete mathematics?
  - the complex field vs. the set of complex numbers
  - identifying formally different but isomorphic structures



# Number sets in current MML

- Initially real numbers were defined axiomatically (article AXIOMS)
  - it allowed users to develop a lot of mathematics right away
  - only in 1998 Trybulec and Bancerek formalized a set theory-based model of real numbers based on a variant of Dedekind cuts
  - example of '*experience, not only doctrine*' ideology
- Properties of the sets NAT, INT, RAT, REAL, and COMPLEX best realized as adjectives natural, integer, rational, real and complex for the type number (synonym to object)
- Less used extensions ExtREAL and QUATERNION are not automated in the system





# Formalization-driven MML refactoring

- Formalizing a book '*A Compendium of Continuous Lattices*' (CCL) by Gierz, Hofmann, Keimel, Lawson, Mislove, and Scott
  - a comprehensive piece of mathematics
  - it stimulated an extensive MML development of lattice theory (in algebraic relational and topological sense)
  - collaborative work involved sixteen authors and resulted in the production of fifty-seven Mizar articles

